Modeling collusion as an informed principal problem*

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Abstract

In this paper we address the question of collusion in mechanisms under asymmetric information by assuming that one of the colluding parties offers a side contract to the other one. We develop a methodology to analyze collusion as an informed principal problem. We show that if collusion occurs after the agents accept or reject the principal's offer, the dominant-strategy implementation of the optimal contract without collusion is collusion proof. In the second part of the paper, we look at a different timing, assuming that the agents' decision to accept or reject the principal's offer is taken after collusion, so the agents share their private information before accepting the principal's offer. On the other hand, we assume that the collusion offer includes a punishment strategy, to be used whenever the other agent rejects the side contract. We establish the conditions that have to be satisfied for a contract to be collusion proof and we show that the optimal contract without collusion is no longer collusion proof. The optimal collusion proof contract is asymmetric, both in transfers and in quantities.

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1 Introduction

In a one-principal-multi-agent environment, the Revelation Principle states that, in the absence of collusion and in a complete contract framework, the optimal allocation can be reached by a centralized organization in which the principal contracts simultaneously with each agent. Any implementable allocation can be replicated by a mechanism in which agents are asked to announce their private information and have incentives to tell the truth. Under the condition that agents tell the truth, the principal extracts as much rent as possible from the agents. The possibility of collusion, however, may modify the set of achievable outcomes for the principal, because the agents may have incentives to coordinate themselves in order to undo the rent extraction. Knowing this, the principal has to find the optimal response to the possibility of collusion.

Collusion seems quite possible in many classical examples in mechanism design. In an auction, for instance, the mechanism designer exacerbates competition between agents in order to obtain a high price for the good. Bidders may have incentives to coordinate their bids and act as a single agent, in order to bid as low as possible. In a firm in which agents work together, it seems quite easy for the agents to coordinate their actions and take advantage of the firm's offer. Nevertheless, collusion may not be an easy task. In the first place, there is a problem of enforcement. Collusion is in general an illegal agreement between the agents, so it is difficult to imagine a collusion agreement being enforced by a court of justice. Therefore, either there is some reliance on reputation or repeated interaction to justify that the colluding parties comply with the agreement, or people, for whatever reason, stick to their promise or word of honor. Second, even if we let aside this question, the asymmetries of information among the agents may introduce frictions at the time of signing an agreement. Indeed, if information is symmetric, any bargaining process will maximize the agents' joint utility and allocate the benefits in some particular way, depending on the distribution of the bargaining power. When information is asymmetric, each agent may want to conceal his private information in order to increase his own utility, and this could go against the maximization of joint utility. The relevant question is, then, what collusion under asymmetric information can achieve.

We consider an organization composed by three parties: a principal, and two agents. Given a contract offered by the principal, the agents can agree on a collusion contract. We assume that one of the agents has all the bargaining power at the collusion stage and makes an enforceable take-it-or-leave-it offer to the other agent (to call this contract we will use the words collusion contract and side contract interchangeably).

Laffont and Martimort (1997, 2000) develop a methodology to analyze the case of optimal contracting when agents can sign side contracts under asymmetric information. They assume that an uninformed third party, interested in the maximization of joint utility, organizes side contracting between agents. By doing so, they avoid looking at the problem of bargaining under asymmetric information. The advantage of this methodology is that the revelation principle applies at the collusion stage.[1] On the other hand, this minimizes the frictions coming from the asymmetry of information, making the coalition of agents very powerful against the principal. Using this methodology, they show that when agents' private information is statistically independent the optimal contract without collusion is implementable even when collusion is possible. This is no longer true when there is correlation between the agents' private information.

Using the methodology developed by Laffont and Martimort, Dequiedt (2002) examines the question of collusion under asymmetric information with independent types, but assuming that agents accept the principal's offer after having agreed on a collusion contract. Looking at the case of a private value auction, he shows that if quantity transfers are not feasible at the collusion stage, the principal can benefit by introducing asymmetric mechanisms, because the presence of asymmetries introduces frictions among the colluding parties. However, when quantity reallocations are feasible, the collusive power is reinforced and the optimal auction is symmetric.

Another piece of literature related to our paper concerns the mechanism design by an informed party. This is exactly what we observe at the collusion stage, where a privately informed agent offers a collusion contract to another agent, also privately informed. Myerson (1983) shows that there is always an equilibrium of the informed principal game. Moreover, he states the *inscrutability principle*, which implies that there is no loss of generality in restricting attention to pooling mechanisms, that is, equilibria in which the principal, whatever his type offers the same mechanism to the agent. Maskin and Tirole (1990, 1992) distinguish two different frameworks to analyze the informed principal problem. In the *private value case* the principal's private information is not an argument of the agent's utility function. They show that when this is the case the equilibrium is always Pareto optimal in the sense that it maximizes a weighted sum of the utilities of the different types of principal. This result is no longer true in the *common value case*, in which the principal's private information is an argument of the agent's utility

[1]Maskin and Tirole (1990) have shown that the revelation principle does not apply in the informed principal game. Indeed, in order to show their main result, they have to expand the set of mechanisms, beyond the set of direct revelation mechanisms.

function. In this case, there may be a unique equilibrium or a continuum of equilibria, and equilibria are typically inefficient from the point of view of the different types of principal.

In this paper we address the question of collusion in mechanisms by assuming that one of the colluding parties offers a side contract to the other one. Collusion then has to be analyzed as an informed principal problem. We assume that the principal is interested in a final good, that is produced using two inputs and each of the agents produces one input. Agents are privately informed about their marginal cost of production and their private information is independent. Agents are perfectly symmetric except for the fact that one of them has all the bargaining power at the collusion stage. As in Laffont and Martimort (1997, 2000), the Revelation Principle has to be replaced by a Collusion-Proofness Principle, which allows us to restrict attention to mechanisms that do not leave any scope for collusion. We show that if collusion occurs after the agents accept or reject the principal's offer, then the optimal contract without collusion (the second-best contract) is collusion-proof, as in Laffont and Martimort (1998). This shows that in this particular case, the use of a third party gives a good approximation of what the principal can obtain when collusion is actually organized by one of the colluding parties. Moreover, we show that even if we give more power to the coalition by assuming that one agent can commit to a punishment strategy if the other agent refuses to collude, the result is not modified: the optimal contract without collusion is still collusion-proof. The difference is that the implementation is achieved through an asymmetric contract.

In the second part of the paper, we consider a different timing, assuming that the agents' decision of whether to accept or reject the principal's offer is taken after collusion as in Dequiedt (2002). Very often there is some delay between the principal's offer and the agents' agreement, which may be used to collude against the principal. This makes this analysis particularly relevant. The most obvious example is the case of auctions. The new feature of this model is that the agents share their private information before accepting the principal's offer, meaning that they know each other's type when the participation decision is taken. Furthermore, we assume that the collusion offer includes a punishment strategy, to be used whenever the other agent rejects the collusion contract. We establish the conditions that have to be satisfied for a contract to be collusion-proof and we show that collusion becomes effective. This means that the second-best is no longer implementable, because it is not collusion-proof. Whenever the principal wants to implement the second-best contract, the agents find a way to undo the principal's offer and increase their utility. However, the presence of asymmetries of information within the coalition implies that collusion is not always efficient. The agents, even coordinating their actions, cannot maximize the sum of their utilities. The principal has to modify her offer with respect to the second-best in order to account for the collusion constraints. Special incentives have to be provided to the agent who has the bargaining power at the collusion stage. To do so, the principal offers an asymmetric contract in which the expected utility of the agent who offers the collusion contract is always higher than that of the other agent. If we were to analyze collusion using the methodolgy developed by Laffont and Martimort, the result would have been that the second-best contract is still implementable even when the punishment is feasible. The reason for such a result is that all the gains from collusion are obtained through side payments between the colluding parties. However, when the third party maximizes joint utility, side payments do not matter. Therefore, we should allow the third party to maximize a weighted sum of the agents' utilities in order to make a more flexible analysis. Moreover, the relative weight given to each agent might be seen as a measure of his bargaining power.

At a first sight, it could seem that contracting with two agents who can collude is equivalent to contracting with a single agent who produces both inputs with a total production cost equal to the sum of the costs of the two agents. However, in our context the principal can do better than in the case in which she faces such a single agent. The reason is that the agent who offers the collusion contract (for instance, agent i) is willing to forgo some efficiency in collusion in order to decrease the informational rent that he is obliged to give up to elicit agent j's private information. When agent j's production cost is high, the coalition does not maximize joint utility, but a "virtual" joint utility, in which the virtual (aggregate) cost is higher than the total production cost because it includes the (expected) cost of the informational rent from i to j. The difference between the virtual cost and the production cost is a measure of the frictions in side contracting created by asymmetric information. In fact, the principal behaves as if she were facing a single agent with virtual costs rather than true (production) costs.

The paper is organized as follows. In Section 2 we describe the main features of the model and we derive a weak collusion-proofness principle that states that the principal can restrict attention to mechanisms for which no collusion is an equilibrium. In Section 3 we assume that the agents collude after having accepted the principal's offer. We provide some conditions for a mechanism to be weakly collusion-proof and we show that the optimal contract without collusion is weakly collusion-proof. In Section 4 we assume that collusion occurs before the agents accept or reject the contract and that the agent who offers the side contract commits to a punishment strategy whenever the other agent rejects his offer. We first show that the agent who offers the side contract always has incentives to set a punishment strategy independent of his type. We give conditions for a contract to be collusion-proof in this context and we show that the optimal contract without collusion is no longer implementable. We characterize the optimal collusion-proof mechanism. We discuss some assumptions of the model in Section 5. Finally, we conclude in Section 6.

2 The model

We consider an organization composed of three agents. A principal (P) produces a final good and contracts with two input suppliers (A^1 and A^2). Agent A^k's contribution to a quantity q of the final good is q^k. We assume a Leontief production technology: $q = q^1 = q^2$.

The principal obtains a monetary revenue from selling the output equal to $S(q)$ and pays a monetary transfer of t^k to A^k. So the principal's total profit is

$$W = S(q) - \left(t^1 + t^2\right).$$

We assume that the function $S(\cdot)$ is increasing and concave and satisfies the Inada conditions: $S'(0) = +\infty$ and $S'(+\infty) = 0$.

Agent k receives a monetary payment t^k and incurs a linear production cost $\theta^k q^k$ when producing q^k units. So A^k's total utility is

$$u^k = t^k - \theta^k q^k.$$

Agent k's marginal cost, θ^k, is his private information. We assume that marginal costs are independent and identically distributed, drawn from a common knowledge distribution with support $\Theta = \{\theta_1, \theta_2\}$ $(\theta_1 < \theta_2)$ $\Delta\theta = \theta_2 - \theta_1$, and we call ν_i the probability of observing θ_i: $\nu_i = \Pr\left(\theta^k = \theta_i\right)$, $\nu_1 + \nu_2 = 1$. We assume that at all the optimal solutions the principal's profit is positive, so it is never optimal to shut down production, even if both agents turn out to be high-cost.

We depart from the previous literature in assuming that at the collusion stage, it is an informed party, A^1, who makes the offer, so the way to analyze this problem is to look at the informed principal literature. For a given offer by the principal, a collusion contract is a manipulation of reports function, ϕ, and side transfers, y, from A^1 to A^2.

We need to introduce some definitions. For a direct revelation mechanism, define the null collusion contract as the contract in which there is no manipulation of reports and side transfers are null: $\phi\left(\widehat{\theta}^1, \widehat{\theta}^2\right) = \left(\widehat{\theta}^1, \widehat{\theta}^2\right)$

and $y\left(\widehat{\theta}^1, \widehat{\theta}^2\right) = 0$ for any announcements made at the collusion stage. We denote by $\emptyset$ the null contract.

Definition 1 *A direct revelation mechanism m^P is* weakly collusion-proof *if there exists one equilibrium of the collusion game in which agent* 1*'s payoff, whatever his type, is the same as with the null collusion contract.*

Definition 2 *A direct revelation mechanism m^P is* strongly collusion-proof *if it is weakly collusion-proof and in any equilibrium of the collusion game, payoffs are the same as in the absence of collusive opportunities.*

Myerson (1983) has shown that there is always at least one equilibrium of the informed principal game. So, for any offer m^P there is at least one equilibrium of the collusion game. In order to analyze the outcome of the collusion game, we invoke Myerson's inscrutability principle (Myerson, 1983), which states that, when looking at the informed principal game, there is no loss of generality in restricting attention to pooling equilibria. This simplifies things a lot, because it means that on the equilibrium path, the collusive offer by agent 1 does not modify the beliefs of agent 2.

We are interested in the best contract for P, knowing that collusion is possible between the two agents. To do that, we prove a weak collusion-proofness principle.

Proposition 1 Weak collusion-proofness principle. *There is no loss of generality in restricting attention to mechanisms that are weakly collusion-proof within the class of direct revelation mechanisms.*

Proof. We sketch the proof, as it follows the lines of Faure-Grimaud, Laffont and Martimort (2002). Suppose P offers a direct revelation mechanism m^P that is not weakly collusion-proof. This means that in any equilibrium of the collusion game, the collusion contract C is different from the null contract. Choose any of those equilibria, i.e. C^* that specifies a manipulation of report function ϕ^* and side transfers y^*. Because this contract is an equilibrium, it satisfies A^1's and A^2's incentive compatibility constraints. Furthermore, because A^2 accepts C^*, it gives him more expected utility than m^P.

Suppose instead that the principal offers a new mechanism $\widetilde{m}^P = m^P \circ C^*$. We claim that if this mechanism is offered, there is one equilibrium of the collusion game in which A^1 offers the null contract. Suppose not. Then, there is another collusion contract, $C' \neq \emptyset$, such that for any out-of-equilibrium beliefs about A^1's type, $\widehat{\nu}$, and for any corresponding out-of-equilibrium payoffs for A^1, $\widehat{u}^1$, for some $k \in \{1, 2\}$, type k of agent 1 has more utility than under the null contract. But then, C^* could not have been an equilibrium in the first place, because the contract $C'' = C^* \circ C'$ gives more utility to type k agent 1 *whatever the out-of-equilibrium beliefs*. Therefore, type k agent 1 would have offered C'' rather than C^*, a contradiction. ■

The idea is that the principal can always eliminate any collusion that could occur by including in her offer the outcome of the collusion. However, the collusion-proofness principle is only weak, because there may be multiple equilibria of the collusion game. There are two sources of multiplicity of equilibria. First, Myerson (1983) and Maskin and Tirole (1992) have proved that, in general, the equilibrium of the informed principal game is not unique. This is a general property of signaling models, although the problem is less important in the informed principal game than in other signaling games, because in the former it is possible to determine a lower bound for the principal's payoff. Second, the conditions for a mechanism to be weakly collusion-proof depend on the outcome of the game in the (out-of-equilibrium) event in which agent 2 rejects the side contract, because this is what determines the status quo at the collusion stage. This outcome, in turn, depends on the out-of-equilibrium beliefs of agent 1 about agent 2's type following a rejection. An usual assumption in the literature on collusion is that beliefs are passive, in the sense that the beliefs following a rejection of the side contract are equal to the prior beliefs.[2]

In contrast with previous collusion-proofness results, we cannot ensure that actually collusion would not happen given a weakly collusion-proof mechanism *even if we restricted ourselves to passive beliefs*. Indeed, due to the multiplicity of equilibria of the informed principal game our collusion-proofness principle is only weak for passive beliefs and therefore, at the collusion stage other equilibria different from the null contract could be selected. That a strong version of the principle cannot be proved is straightforward, because whenever the principal's offer m^P is weakly collusion-proof but fails to be strongly collusion-proof, the principal may actually obtain the payoff corresponding to m^P if the non-collusive equilibrium is selected. The collusion-proofness principle implies that in order to characterize the principal's feasible set we need to find necessary and sufficient conditions for an allocation to be weakly collusion-proof.

[2]See, for instance, Laffont and Martimort (1997), (1998), (2000) and Faure-Grimaud, Laffont and Martimort (2001).

3 Ex post collusion

In this section, we follow the approach in Laffont and Martimort (1997, 2000), in the sense that we assume that collusion, if any, occurs after the decision to accept or reject the principal's offer.

The timing is as follows:

1. P offers a mechanism, m^P, to A^1 and A^2. m^P determines a quantity to be produced and a vector of transfers for the two agents as functions of their reports: $\left\{q\left(\widetilde{\theta}^1,\widetilde{\theta}^2\right), t^1\left(\widetilde{\theta}^1,\widetilde{\theta}^2\right), t^2\left(\widetilde{\theta}^1,\widetilde{\theta}^2\right)\right\}$ where $\widetilde{\theta}^k$ is A^k's announcement of his own marginal cost in m^P.[3]

2. A^1 and A^2 simultaneously accept or reject P's offer. If one of them rejects, the game is over and everyone obtains 0. If they both accept, they go to stage 3.

3. A^1 offers a collusion contract, C, to A^2. C determines a manipulation of reports (ϕ) and a transfer (y) from A^1 to A^2 as functions of both agent's reports: $\left\{\phi\left(\widehat{\theta}^1,\widehat{\theta}^2\right), y\left(\widehat{\theta}^1,\widehat{\theta}^2\right)\right\}$, where $\widehat{\theta}^k$ is A^k's announcement of his own marginal cost in C.

4. A^2 accepts or rejects A^1's offer. If A^2 rejects the offer, A^1 and A^2 go to stage 6. If A^2 accepts A^1's offer, they go to stage 5.

5. A^1 and A^2 simultaneously announce their types to each other.

6. A^1 and A^2 simultaneously announce their types to the principal (according to the function ϕ if they had agreed on a collusion contract).

7. Production is carried-out and transfers are paid.

The problem that we have to analyze corresponds to the common value framework as defined by Maskin and Tirole (1990, 1992), because A^2's status quo level at the collusion stage is given by the utility he would obtain under m^P, whenever he rejects the side contract, which, in turn, is a function of A^1's private information. Although there is a problem of multiplicity of equilibria,

[3] We are here restricting ourselves to mechanisms in which each agent announces only his own type. However, the principal knows that, if there is collusion, agents will know each other's type when playing the grand mechanism. Therefore, a more general mechanism would be to make both agents announce both types. In this case, the message space of each agent is $\Theta \times \{\emptyset, \Theta\}$, where announcing $\emptyset$ means that there was no collusion. Because the agents can always manipulate information and announce $(\emptyset, \emptyset)$, the principal cannot gain with such a mechanism (see Laffont and Martimort, 1997).

Maskin and Tirole (1992) have shown that there is a lower bound to the payoff of the agent who offers the contract that can be easily characterized.

3.1 Some efficiency concepts

Collusion-proofness implies some kind of efficiency of the null contract at the collusion stage. Exactly which kind of efficiency is something that we will determine afterwards. For the time being, let us concentrate on some efficiency definitions, borrowed from Maskin and Tirole (1992) and adapted to this particular context.

Definition 3 *An allocation* $\left(\overline{\phi}_{k\ell}, \overline{y}_{k\ell}\right)_{\substack{k=1,2\\ \ell=1,2}}$ *is* weakly interim efficient *(WIE) if (a) it is interim incentive compatible for* A^1 *and (b) there is no allocation satisfying (a) that,* regardless of A^1's type, *is incentive compatible for* A^2 *and gives* A^2 *at least as much utility. That is, a WIE allocation is, for some vector of positive weights,* $\{w_k\}_{k=1,2}$ *a solution to problem I defined as*[4]

$$\max_{(\phi_{k\ell}, y_{k\ell})_{\substack{k=1,2\\ \ell=1,2}}} \sum_k w_k \sum_\ell \nu_\ell \left(t^1\left(\phi_{k\ell}\right) - y_{k\ell} - \theta_k q\left(\phi_{k\ell}\right)\right)$$

subject to

$$\sum_\ell \nu_\ell \left(t^1\left(\phi_{i\ell}\right) - y_{i\ell} - \theta_i q\left(\phi_{i\ell}\right)\right) \geq$$
$$\sum_\ell \nu_\ell \left(t^1\left(\phi_{i'\ell}\right) - y_{i'\ell} - \theta_i q\left(\phi_{i'\ell}\right)\right) \quad \forall (i, i') \in \{1,2\}^2,$$
$$t^2\left(\phi_{i\ell}\right) + y_{i\ell} - \theta_\ell q\left(\phi_{i\ell}\right) \geq t^2\left(\phi_{i\ell'}\right) + y_{i\ell'} - \theta_\ell q\left(\phi_{i\ell'}\right) \quad \forall (i, \ell, \ell') \in \{1,2\}^3,$$
$$t^2\left(\phi_{i\ell}\right) + y_{i\ell} - \theta_\ell q\left(\phi_{i\ell}\right) \geq t^2\left(\overline{\phi}_{i\ell}\right) + \overline{y}_{i\ell} - \theta_\ell q\left(\overline{\phi}_{i\ell}\right) \quad \forall (i, \ell) \in \{1,2\}^2.$$

Definition 4 *An allocation* $\left(\widehat{\phi}_{k\ell}, \widehat{y}_{k\ell}\right)_{\substack{k=1,2\\ \ell=1,2}}$ *is a* Rothschild-Stiglitz-Wilson *(RSW) allocation relative to the status quo allocation* $\left(\phi^0_{k\ell}, y^0_{k\ell}\right)_{\substack{k=1,2\\ \ell=1,2}}$ *if and only if, for all k:* $(\phi_{k\ell}, y_{k\ell})_{\substack{k=1,2\\ \ell=1,2}} = \left(\widehat{\phi}_{k\ell}, \widehat{y}_{k\ell}\right)_{\substack{k=1,2\\ \ell=1,2}}$*; and* $\forall k$, $\left(\widehat{\phi}_{k\ell}, \widehat{y}_{k\ell}\right)_{\ell=1,2}$ *is*

[4] Because we allow a stochastic manipulation of reports, the function $t^i\left(\phi_{k\ell}\right)$ (resp. $q\left(\phi_{k\ell}\right)$) has to be interpreted as an expectation with respect to the distribution of reports in the grand contract induced by $\phi_{k\ell}$. Similarly, $S\left(q\left(\phi_{k\ell}\right)\right)$ is the expectation of the function S.

obtained as a solution to problem II_k defined as

$$\max_{(\phi_{k\ell},y_{k\ell})_{\substack{k=1,2\\ \ell=1,2}}} \sum_{\ell} \nu_{\ell}\left(t^{1}\left(\phi_{k\ell}\right)-y_{k\ell}-\theta_{k} q\left(\phi_{k\ell}\right)\right)$$

subject to

$$\sum_{\ell} \nu_{\ell}\left(t^{1}\left(\phi_{i\ell}\right)-y_{i\ell}-\theta_{i} q\left(\phi_{i\ell}\right)\right) \geq$$

$$\sum_{\ell} \nu_{\ell}\left(t^{1}\left(\phi_{i'\ell}\right)-y_{i'\ell}-\theta_{i} q\left(\phi_{i'\ell}\right)\right) \quad \forall\left(i, i'\right) \in\{1,2\}^{2}$$

$$t^{2}\left(\phi_{i\ell}\right)+y_{i\ell}-\theta_{\ell} q\left(\phi_{i\ell}\right) \geq t^{2}\left(\phi_{i\ell'}\right)+y_{i\ell'}-\theta_{\ell} q\left(\phi_{i\ell'}\right) \quad \forall\left(i, \ell, \ell'\right) \in\{1,2\}^{3}$$

$$t^{2}\left(\phi_{i\ell}\right)+y_{i\ell}-\theta_{\ell} q\left(\phi_{i\ell}\right) \geq t^{2}\left(\phi_{i\ell}^{0}\right)+y_{i\ell}^{0}-\theta_{\ell} q\left(\phi_{i\ell}^{0}\right) \quad \forall(i, \ell) \in\{1,2\}^{2}.$$

As Maskin and Tirole (1992) have shown, any RSW allocation is WIE and, therefore, incentive compatible for agent 1. Moreover, any WIE allocation is RSW relative to itself. This result will turn out to be important for the characterization of weakly collusion-proof mechanisms.

Definition 5 *An allocation $\left(\overline{\phi}_{k\ell}, \overline{y}_{k\ell}\right)_{\substack{k=1,2\\ \ell=1,2}}$ is* interim efficient relative to beliefs $\widehat{\nu}$ *(IE($\widehat{\nu}$)) if and only if for some vector of positive weights, $\{w_k\}_{k=1,2}$ it is a solution to problem III defined as*

$$\max_{(\phi_{k\ell},y_{k\ell})_{\substack{k=1,2\\ \ell=1,2}}} \sum_{k} w_{k} \sum_{\ell} \nu_{\ell}\left(t^{1}\left(\phi_{k\ell}\right)-y_{k\ell}-\theta_{k} q\left(\phi_{k\ell}\right)\right)$$

subject to

$$\sum_{\ell} \nu_{\ell}\left(t^{1}\left(\phi_{i\ell}\right)-y_{i\ell}-\theta_{i} q\left(\phi_{i\ell}\right)\right) \geq$$

$$\sum_{\ell} \nu_{\ell}\left(t^{1}\left(\phi_{i'\ell}\right)-y_{i'\ell}-\theta_{i} q\left(\phi_{i'\ell}\right)\right) \quad \forall\left(i, i'\right) \in\{1,2\}^{2}$$

$$\sum_{i} \widehat{\nu}_{i}\left(t^{2}\left(\phi_{i\ell}\right)+y_{i\ell}-\theta_{\ell} q\left(\phi_{i\ell}\right)\right) \geq$$

$$\sum_{i} \widehat{\nu}_{i}\left(t^{2}\left(\phi_{i\ell'}\right)+y_{i\ell'}-\theta_{\ell} q\left(\phi_{i\ell'}\right)\right) \quad \forall\left(\ell, \ell'\right) \in\{1,2\}^{2}$$

$$\sum_{i} \widehat{\nu}_{i}\left(t^{2}\left(\phi_{i\ell}\right)+y_{i\ell}-\theta_{\ell} q\left(\phi_{i\ell}\right)\right) \geq$$

$$\sum_{i} \widehat{\nu}_{i}\left(t^{2}\left(\overline{\phi}_{i\ell}\right)+\overline{y}_{i\ell}-\theta_{\ell} q\left(\overline{\phi}_{i\ell}\right)\right) \quad \forall \ell \in\{1,2\}.$$

An important result is that, regardless of his type, agent 1 can always guarantee himself the payoff corresponding to the RSW allocation. Indeed, he can propose the allocation that solves problem II_k. Notice, then, that whatever the beliefs of agent 2 about agent 1's type, A^2 accepts the contract and is truthful, and therefore, A^1's payoff is the RSW payoff. This implies that in order to look for the set of equilibria of the collusion game, only allocations that give at least the RSW payoff to both types of A^1 can be candidates. We will call $\widehat{u}_k^1$ the RSW payoff of type k agent 1.

In order to look for the set of weakly collusion-proof mechanisms, we can restrict attention to (interim) incentive compatible direct revelation mechanisms, because, on the equilibrium path, collusion does not occur.

Proposition 2 *Assume that the RSW allocation is IE relative to some strictly positive beliefs and that the null contract satisfies agent 2's incentive constraints whatever the type of agent 1. Then, a) if the null contract is RSW relative to the status quo allocation, the mechanism m^P is weakly collusion-proof; b) the converse is also true provided that agent 1 is truthful even if agent 2 rejects the side contract.*

Proof. a) If the RSW allocation is IE relative to strictly positive beliefs, the RSW allocation is an equilibrium of the game (Theorem 1* in Maskin-Tirole, 1992). But the RSW allocation is the null contract itself. Therefore, the null contract is an equilibrium of the game and m^P is weakly collusion-proof.

b) Since A^1 can guarantee the RSW payoff, we know that in any equilibrium of the collusion game, any type k A^1 gets $\widetilde{u}_k^1 \geq \widehat{u}_k^1$. Now, because A^1 is truthful on m^P when A^2 rejects the side contract, the status quo payoff of A^2 is the null contract (by assumption, A^2 is truthful whatever his beliefs about A^1's type). Suppose that the null contract is not RSW relative to itself. Then, it is not weakly interim efficient. Now, since the null contract satisfies the ex post incentive compatibility constraints of A^2, it satisfies all the constraints in program I (it satisfies A^1's incentive compatibility constraints because m^P is interim incentive compatible), but it is not RSW, so $\forall k$, $\widehat{u}_k^1 \geq u_k^{10}$, where u_k^{10} is the utility of type k A^1 under the null contract. Moreover, $\exists k$ such that $\widehat{u}_k^1 > u_k^{10}$ because the null contract is not WIE. Thus, the null contract cannot be an equilibrium of the collusion game, because it gives strictly less utility than the RSW allocation to at least one type. Therefore, m^P is not weakly collusion-proof. ■

Proposition 2 gives, then, necessary and sufficient conditions for the grand contract to be weakly collusion-proof, provided that the RSW allocation is interim efficient for some strictly positive beliefs. Loosely, a mechanism is weakly collusion-proof if and only if the null contract corresponding to this mechanism is an RSW allocation in the collusion game. Maskin and Tirole

(1992) describe the set of equilibria of the informed principal game with common values as the set of allocations that are interim incentive compatible for both agents, give more than the outside opportunity to agent 2 and Pareto dominate the RSW allocation from A^1's perspective. This implies that the RSW allocation itself is an equilibrium of the game. Therefore, if the null contract is RSW, it is an equilibrium of the collusion game and the grand contract is weakly collusion-proof. To show the converse is a bit more tricky, because of two reasons. First, there may exist a weakly collusion-proof contract that satisfies the interim incentive constraints for agent 2, but not the ex post incentive constraints. By definition, this contract cannot be an RSW allocation. Second, it may be the case that, when agent 2 rejects the side contract, the original offer, m^P, is not incentive compatible for agent 1 given his new (out-of-equilibrium) beliefs about agent 2's type. This changes agent 2's outside option, which depends on the announcement made by agent 1 to the principal. It could be, then, that the null contract cannot compensate this new outside option, so it cannot be an RSW allocation. There are two obvious cases in which agent 1 is truthful out of the equilibrium path. One is the already mentioned case of passive beliefs. Because the original offer is incentive compatible for the prior beliefs, whenever the out-of-equilibrium beliefs are equal to the priors, agent 1's incentive compatibility constraints are satisfied also out of the equilibrium path. The second case, is the case in which agent 1's incentive compatibility constraints are satisfied whatever the type of agent 2. This case is particularly interesting, because the fact that agent 1 is truthful out of the equilibrium path is independent of the specific out-of-equilibrium beliefs.

3.2 The second-best contract

If collusion between agents is impossible (for instance, non-enforceable), the best the principal can do is to offer the second-best contract, $m^{sb} = \left(t^{1sb}, t^{2sb}, q^{sb}\right)$ that solves problem B, defined as

$$\max_{\left(t^1_{k\ell}, t^2_{k\ell}, q_{k\ell}\right)_{\substack{k=1,2 \\ \ell=1,2}}} \sum_k \sum_\ell \nu_k \nu_\ell \left(S\left(q_{k\ell}\right) - t^1_{k\ell} - t^2_{k\ell}\right)$$

subject to

$$
\begin{aligned}
\sum_k \nu_k \left(t^2_{k1} - \theta_1 q_{k1}\right) &\geq \sum_k \nu_k \left(t^2_{k2} - \theta_1 q_{k2}\right), \\
\sum_k \nu_k \left(t^2_{k2} - \theta_2 q_{k2}\right) &\geq \sum_k \nu_k \left(t^2_{k1} - \theta_2 q_{k1}\right), \\
\sum_\ell \nu_\ell \left(t^1_{1\ell} - \theta_1 q_{1\ell}\right) &\geq \sum_\ell \nu_\ell \left(t^1_{2\ell} - \theta_1 q_{2\ell}\right), \\
\sum_\ell \nu_\ell \left(t^1_{2\ell} - \theta_2 q_{2\ell}\right) &\geq \sum_\ell \nu_\ell \left(t^1_{1\ell} - \theta_2 q_{1\ell}\right),
\end{aligned}
$$

$$
\begin{aligned}
\sum_k \nu_k \left(t^2_{k1} - \theta_1 q_{k1}\right) &\geq 0, \\
\sum_k \nu_k \left(t^2_{k2} - \theta_2 q_{k2}\right) &\geq 0, \\
\sum_\ell \nu_\ell \left(t^1_{1\ell} - \theta_1 q_{1\ell}\right) &\geq 0, \\
\sum_\ell \nu_\ell \left(t^1_{2\ell} - \theta_2 q_{2\ell}\right) &\geq 0.
\end{aligned}
$$

The second-best contract, m^{sb}, is given by

$$
\begin{aligned}
\sum_k \nu_k t^{2sb}_{k2} &= \theta_2 \sum_k \nu_k q^{sb}_{k2}, \\
\sum_\ell \nu_\ell t^{1sb}_{2\ell} &= \theta_2 \sum_\ell \nu_\ell q^{sb}_{2\ell}, \\
\sum_k \nu_k t^{2sb}_{k1} &= \theta_1 \sum_k \nu_k q^{sb}_{k1} + \Delta\theta \sum_k \nu_k q^{sb}_{k2}, \\
\sum_\ell \nu_\ell t^{1sb}_{1\ell} &= \theta_1 \sum_\ell \nu_\ell q^{sb}_{1\ell} + \Delta\theta \sum_\ell \nu_\ell q^{sb}_{2\ell},
\end{aligned}
$$

$$
\begin{aligned}
S'\left(q^{sb}_{11}\right) &= 2\theta_1, \\
S'\left(q^{sb}_{12}\right) &= \theta_1 + \theta_2 + \frac{\nu_1}{\nu_2}\Delta\theta, \\
S'\left(q^{sb}_{21}\right) &= \theta_1 + \theta_2 + \frac{\nu_1}{\nu_2}\Delta\theta, \\
S'\left(q^{sb}_{22}\right) &= 2\theta_2 + 2\frac{\nu_1}{\nu_2}\Delta\theta.
\end{aligned}
$$

Since the agents are both risk neutral, the principal has some degrees of freedom in choosing the transfers, because only the expected transfer given to each agent is determined by the optimal contract (the principal has 4 linear equations to determine 8 transfers). The quantities, however, are uniquely determined in the second-best contract.

One interesting property of the second-best contract is that it can be implemented in dominant strategies (see Laffont and Tirole, 1993, chap. 7 and Mookherjee and Reichelstein, 1992). That is, we can find transfers such that the incentive and participation constraints of agent i are satisfied whatever agent j's type:

$$\begin{aligned} t_{11}^{1ds} &= t_{11}^{2ds} = \theta_1 q_{11}^{sb} + \Delta\theta q_{12}^{sb}, \\ t_{12}^{1ds} &= t_{21}^{2ds} = \theta_1 q_{12}^{sb} + \Delta\theta q_{22}^{sb}, \\ t_{21}^{1ds} &= t_{12}^{2ds} = \theta_2 q_{12}^{sb}, \\ t_{22}^{1ds} &= t_{22}^{2ds} = \theta_2 q_{22}^{sb}. \end{aligned} \tag{1}$$

We are now ready to state the main result of this section.

Proposition 3 *The dominant-strategy implementation of the second-best contract is weakly collusion-proof.*

Proof. By definition, the dominant strategy implementation satisfies A^i's incentive constraints type by type, therefore, it is weakly collusion-proof if and only if at the collusion stage, the null contract is the RSW allocation and it is interim efficient for some strictly positive beliefs (this last point is proved in Proposition 5). By definition, the null contract is RSW relative to itself if it solves

$$\begin{array}{ll}
\max\limits_{(\phi,y)} \sum_k \sum_\ell \nu_\ell \left(t^1\left(\phi_{k\ell}\right) - y_{k\ell} - \theta_k q\left(\phi_{k\ell}\right)\right) & \\
\text{subject to} & \\
t^2\left(\phi_{11}\right) + y_{11} - \theta_1 q\left(\phi_{11}\right) \geq t^2\left(\phi_{12}\right) + y_{12} - \theta_1 q\left(\phi_{12}\right) & (\lambda_{11}) \\
t^2\left(\phi_{12}\right) + y_{12} - \theta_2 q\left(\phi_{12}\right) \geq t^2\left(\phi_{11}\right) + y_{11} - \theta_2 q\left(\phi_{11}\right) & (\lambda_{12}) \\
t^2\left(\phi_{21}\right) + y_{21} - \theta_1 q\left(\phi_{21}\right) \geq t^2\left(\phi_{22}\right) + y_{22} - \theta_1 q\left(\phi_{22}\right) & (\lambda_{21}) \\
t^2\left(\phi_{22}\right) + y_{22} - \theta_2 q\left(\phi_{22}\right) \geq t^2\left(\phi_{21}\right) + y_{21} - \theta_2 q\left(\phi_{21}\right) & (\lambda_{22}) \\
t^2\left(\phi_{11}\right) + y_{11} - \theta_1 q\left(\phi_{11}\right) \geq \Delta\theta q_{12}^{sb} & (\mu_{11}) \\
t^2\left(\phi_{12}\right) + y_{12} - \theta_2 q\left(\phi_{12}\right) \geq \Delta\theta q_{22}^{sb} & (\mu_{12}) \\
t^2\left(\phi_{21}\right) + y_{21} - \theta_1 q\left(\phi_{21}\right) \geq 0 & (\mu_{21}) \\
t^2\left(\phi_{22}\right) + y_{22} - \theta_2 q\left(\phi_{22}\right) \geq 0 & (\mu_{22}) \\
\sum_\ell \nu_\ell \left(t^1\left(\phi_{1\ell}\right) - y_{1\ell} - \theta_1 q\left(\phi_{1\ell}\right)\right) \geq \sum_\ell \nu_\ell \left(t^1\left(\phi_{2\ell}\right) - y_{2\ell} - \theta_1 q\left(\phi_{2\ell}\right)\right) & (\gamma_1) \\
\sum_\ell \nu_\ell \left(t^1\left(\phi_{2\ell}\right) - y_{2\ell} - \theta_2 q\left(\phi_{2\ell}\right)\right) \geq \sum_\ell \nu_\ell \left(t^1\left(\phi_{1\ell}\right) - y_{1\ell} - \theta_2 q\left(\phi_{1\ell}\right)\right) & (\gamma_2)\,.
\end{array}$$

At the null contract, we have $\phi_{k\ell} = (\theta_k, \theta_\ell)\ \forall k, \ell,\ y_{k\ell} = 0\ \forall k, \forall \ell$ and $\lambda_{12} = \lambda_{22} = \gamma_2 = 0$, because the incentive constraints for a high-cost agent

i are slack. Moreover, all the constraints of the problem above are satisfied, so we just need to show that it maximizes the objective function. Taking derivatives with respect to $y_{k\ell}$, we obtain that an RSW allocation satisfies the first order conditions:

$$\begin{aligned}
\mu_{11} &= \nu_1(1+\gamma_1) - \lambda_{11} \geq 0, \\
\mu_{21} &= \nu_1(1-\gamma_1) - \lambda_{21} \geq 0, \\
\mu_{12} &= \nu_2(1+\gamma_1) + \lambda_{11} \geq 0, \\
\mu_{22} &= \nu_2(1-\gamma_1) + \lambda_{21} \geq 0.
\end{aligned} \tag{2}$$

Optimizing with respect to ϕ_{ij} and using (2), we obtain that the null contract is RSW if, for some $\mu_{ij} \geq 0$, $\lambda_{11} \geq 0$, $\lambda_{21} \geq 0$ and $\gamma_1 \geq 0$,

$$\begin{aligned}
(\theta_1,\theta_1) &\in \arg\max_{\widetilde{\phi}} \left\{ t^1\left(\widetilde{\phi}\right) + t^2\left(\widetilde{\phi}\right) - 2\theta_1 q\left(\widetilde{\phi}\right) \right\}, \\
(\theta_1,\theta_2) &\in \arg\max_{\widetilde{\phi}} \left\{ t^1\left(\widetilde{\phi}\right) + t^2\left(\widetilde{\phi}\right) - (\theta_1+\theta_2) q\left(\widetilde{\phi}\right) - \tfrac{\lambda_{11}}{\nu_2(1+\gamma_1)} \Delta\theta q\left(\widetilde{\phi}\right) \right\}, \\
(\theta_2,\theta_1) &\in \arg\max_{\widetilde{\phi}} \left\{ t^1\left(\widetilde{\phi}\right) + t^2\left(\widetilde{\phi}\right) - (\theta_1+\theta_2) q\left(\widetilde{\phi}\right) - \tfrac{\gamma_1}{1-\gamma_1} \Delta\theta q\left(\widetilde{\phi}\right) \right\}, \\
(\theta_2,\theta_2) &\in \arg\max_{\widetilde{\phi}} \left\{ t^1\left(\widetilde{\phi}\right) + t^2\left(\widetilde{\phi}\right) - 2\theta_2 q\left(\widetilde{\phi}\right) - \tfrac{\lambda_{21}+\nu_2\gamma_1}{\nu_2(1-\gamma_1)} \Delta\theta q\left(\widetilde{\phi}\right) \right\},
\end{aligned} \tag{3}$$

or, $\forall (k,\ell) \in \{1,2\}^2$, the following collusion constraints are satisfied:

$$\begin{aligned}
& t_{11}^{1ds} + t_{11}^{2ds} - 2\theta_1 q_{11}^{sb} \geq t_{k\ell}^{1ds} + t_{k\ell}^{2ds} - 2\theta_1 q_{k\ell}^{sb}, \\
& t_{12}^{1ds} + t_{12}^{2ds} - (\theta_2+\theta_1) q_{12}^{sb} - \frac{\lambda_{11}}{\nu_2(1+\gamma_1)} \Delta\theta q_{12}^{sb} \\
\geq\ & t_{k\ell}^{1ds} + t_{k\ell}^{2ds} - (\theta_2+\theta_1) q_{k\ell}^{sb} - \frac{\lambda_{11}}{\nu_2(1+\gamma_1)} \Delta\theta q_{k\ell}^{sb}, \\
& t_{21}^{1ds} + t_{21}^{2ds} - (\theta_2+\theta_1) q_{21}^{sb} - \frac{\gamma_1}{1-\gamma_1} \Delta\theta q_{21}^{sb} \\
\geq\ & t_{k\ell}^{1ds} + t_{k\ell}^{2ds} - (\theta_2+\theta_1) q_{k\ell}^{sb} - \frac{\gamma_1}{1-\gamma_1} \Delta\theta q_{k\ell}^{sb}, \\
& t_{22}^{1ds} + t_{22}^{2ds} - 2\theta_2 q_{22}^{sb} - \frac{\lambda_{21}+\nu_2\gamma_1}{\nu_2(1-\gamma_1)} \Delta\theta q_{22}^{sb} \\
\geq\ & t_{k\ell}^{1ds} + t_{k\ell}^{2ds} - 2\theta_2 q_{k\ell}^{sb} - \frac{\lambda_{21}+\nu_2\gamma_1}{\nu_2(1-\gamma_1)} \Delta\theta q_{k\ell}^{sb}.
\end{aligned}$$

Using (1), we have that the dominant-strategy implementation of the second-best satisfies all the collusion constraints for $\lambda_{11} = \lambda_{21} = \gamma_1 = 0$.

Therefore, the null contract is RSW relative to itself and the second-best contract is weakly collusion-proof, according to Proposition 2. ■

According to Proposition 3, collusion does not have any effect on the principal's offer. The principal can still implement the second-best contract in dominant strategies even if agents can collude, by appropriately choosing the values of the multipliers. Notice that this implementation of the second-best contract is symmetric, even though agents are actually asymmetric, because at the collusion stage all the bargaining power belongs to agent 1. This is also the result found in Laffont and Martimort (1997) when they allow for non-anonimous contracts. The difference with their paper is not in the result, but in the methodology. We provide here some insights on how one should look at the problem of collusion under asymmetric information, when it is organized by one of the colluding parties. This is a first step for the analysis of collusion as a bargaining process under asymmetric information.

The dominant-strategy implementation of the second-best contract is weakly collusion-proof, so there is one equilibrium in which collusion does not occur. However, given the multiplicity of equilibria of the collusion game, this does not mean that collusion will not actually happen. Indeed, it may be the case that at the collusion stage there be other equilibria, and any equilibrium could be selected. Therefore, an interesting question is whether the null contract could be the unique equilibrium at the collusion stage. We can easily eliminate one source of multiplicity of equilibria.

Remark 1 *The dominant-strategy implementation of the second-best contract is weakly collusion-proof whatever the out-of-equilibrium beliefs following a rejection of agent 2.*

The idea is that because the contract is implemented in dominant strategies, the incentive constraints of agent 1 when there is no collusion are satisfied *for any beliefs about agent 2's type.* Therefore, suppose agent 2 rejects the side contract. Then, no matter what agent 1 infers from this action, he has always incentives to tell the truth. Thus, agent 1 announces θ_i if he is actually type i, what happens with probability ν_i. So, from type j agent 2's point of view, the incentive constraints write

$$\sum_i \nu_i \left(t_{ij}^2 - \theta_j q_{ij}\right) \geq \sum_i \nu_i \left(t_{i\ell}^2 - \theta_j q_{i\ell}\right),$$

which are satisfied, because the contract is incentive compatible in dominant strategies also for agent 2.

Proposition 4 *For given out-of-equilibrium beliefs following a rejection, if the null contract is an RSW allocation relative to the status quo and is interim efficient relative to the prior beliefs, then the null contract is the unique equilibrium of the collusion game.*

Proof. Because the RSW allocation is interim efficient relative to the prior beliefs (which are strictly positive), then the set of equilibrium allocations is the set of allocations that satisfy the interim incentive compatibility constraints of the two agents and the interim participation constraints of agent 2 and Pareto dominate (from agent 1's view point) the RSW allocation. Even if the RSW allocation is not unique, the RSW payoffs for A^1 are. Now, because the null contract is an RSW allocation and is interim efficient relative to the prior beliefs, in any equilibrium of the collusion stage, A^1's payoff is the RSW payoff. Therefore, A^1 cannot do strictly better by offering a non-null collusion contract and, thus, no collusion occurs. ■

When the RSW allocation is interim efficient relative to the prior beliefs, the set of equilibria of the informed principal game shrinks to the RSW allocation. By definition of interim efficiency, no other incentive compatible allocation dominates the RSW allocation. Therefore, once agent 1's beliefs about agent 2's type following a rejection of the side contract are fixed (therefore, agent 2's status quo payoff is also fixed), if the null contract is RSW and interim efficient relative to the prior beliefs, it has to be the unique equilibrium.

Given Proposition 4, the second-best contract is strongly collusion-proof if it is weakly collusion proof and the null contract is interim efficient relative to the prior beliefs. By definition, the null contract is interim efficient relative to beliefs $\widehat{\nu}$ if, for some vector of strictly positive weights (w_1, w_2) it solves

$$
\begin{array}{ll}
\max\limits_{(\phi_{k\ell}, y_{k\ell})} \sum_k \sum_\ell w_k \nu_\ell \left(t^1(\phi_{k\ell}) - y_{k\ell} - \theta_k q(\phi_{k\ell})\right) & \\
\text{subject to} & \\
\sum_k \widehat{\nu}_k \left(t^2(\phi_{k1}) + y_{k1} - \theta_1 q(\phi_{k1})\right) \geq \sum_k \widehat{\nu}_k \left(t^2(\phi_{k2}) + y_{k2} - \theta_1 q(\phi_{k2})\right) & \left(\widehat{\lambda}_1\right) \\
\sum_k \widehat{\nu}_k \left(t^2(\phi_{k2}) + y_{k2} - \theta_2 q(\phi_{k2})\right) \geq \sum_k \widehat{\nu}_k \left(t^2(\phi_{k1}) + y_{k1} - \theta_2 q(\phi_{k1})\right) & \left(\widehat{\lambda}_2\right) \\
\sum_k \widehat{\nu}_k \left(t^2(\phi_{k1}) + y_{k1} - \theta_1 q(\phi_{k1})\right) \geq \sum_k \widehat{\nu}_k \Delta\theta q^{sb}_{k2} & (\widehat{\mu}_1) \\
\sum_k \widehat{\nu}_k \left(t^2(\phi_{k2}) + y_{k2} - \theta_2 q(\phi_{k2})\right) \geq 0 & (\widehat{\mu}_2) \\
\sum_\ell \nu_\ell \left(t^1(\phi_{1\ell}) - y_{1\ell} - \theta_1 q(\phi_{1\ell})\right) \geq \sum_\ell \nu_\ell \left(t^1(\phi_{2\ell}) - y_{2\ell} - \theta_1 q(\phi_{2\ell})\right) & (\widehat{\gamma}_1) \\
\sum_\ell \nu_\ell \left(t^1(\phi_{2\ell}) - y_{2\ell} - \theta_2 q(\phi_{2\ell})\right) \geq \sum_\ell \nu_\ell \left(t^1(\phi_{1\ell}) - y_{1\ell} - \theta_2 q(\phi_{1\ell})\right) & (\widehat{\gamma}_2)\,.
\end{array}
$$

Keeping the assumption that agent 1 is truthful also out of the equilibrium path (which is true for the dominant-strategy implementation), we know that the null contract is RSW relative to itself, so, it satisfies all the constraints for any vector $\widehat{\nu}$. Moreover, $\widehat{\lambda}_2 = \widehat{\gamma}_2 = 0$. So we only need to check that it maximizes the objective function for the prior beliefs.

Proposition 5 *If the second-best contract is offered and the null contract is RSW relative to itself, then, the null contract is interim efficient relative to the prior beliefs.*

Proof. Suppose without loss of generality that $w_1 + w_2 = 1$. From the first order conditions with respect to $y_{k\ell}$, the null contract is interim efficient relative to beliefs $\widehat{\nu}$ if

$$\begin{aligned} \widehat{\gamma}_1 &= \widehat{\nu}_1 - w_1 \geq 0, \\ \widehat{\mu}_2 &= 1 - \widehat{\mu}_1 \geq 0, \\ \widehat{\lambda}_1 &= \nu_1 - \widehat{\mu}_1 \geq 0. \end{aligned} \tag{4}$$

So, it is interim efficient relative to the prior beliefs if the conditions are satisfied for $\widehat{\nu}_i = \nu_i$.

Optimizing with respect to ϕ_{ij} and using (4), the following conditions have to be satisfied for some $\widehat{\mu}_1 \in [0, \nu_1]$, $w_1 \in (0, \nu_1]$

$$\begin{aligned} (\theta_1, \theta_1) &\in \arg\max_{\widetilde{\phi}} \left\{ t^1\left(\widetilde{\phi}\right) + t^2\left(\widetilde{\phi}\right) - 2\theta_1 q\left(\widetilde{\phi}\right) \right\}, \\ (\theta_1, \theta_2) &\in \arg\max_{\widetilde{\phi}} \left\{ t^1\left(\widetilde{\phi}\right) + t^2\left(\widetilde{\phi}\right) - (\theta_2 + \theta_1) q\left(\widetilde{\phi}\right) - \tfrac{\nu_1 - \widehat{\mu}_1}{\nu_2} \Delta\theta q\left(\widetilde{\phi}\right) \right\}, \\ (\theta_2, \theta_1) &\in \arg\max_{\widetilde{\phi}} \left\{ t^1\left(\widetilde{\phi}\right) + t^2\left(\widetilde{\phi}\right) - (\theta_2 + \theta_1) q\left(\widetilde{\phi}\right) - \tfrac{\nu_1 - w_1}{\nu_2} \Delta\theta q\left(\widetilde{\phi}\right) \right\}, \\ (\theta_2, \theta_2) &\in \arg\max_{\widetilde{\phi}} \left\{ t^1\left(\widetilde{\phi}\right) + t^2\left(\widetilde{\phi}\right) - 2\theta_2 q\left(\widetilde{\phi}\right) - \tfrac{2\nu_1 - \widehat{\mu}_1 - w_1}{\nu_2} \Delta\theta q\left(\widetilde{\phi}\right) \right\}. \end{aligned} \tag{5}$$

By assumption, the null contract is RSW relative to itself, so conditions (3) are satisfied. Moreover, (3) imply (5) whenever

$$\begin{aligned} \widehat{\mu}_1 &= \frac{\nu_1 (1 + \gamma_1) - \lambda_{11}}{1 + \gamma_1} \geq 0, \\ w_1 &= \frac{\nu_1 - \gamma_1}{1 - \gamma_1} > 0, \\ \lambda_{21} &= \lambda_{11} \frac{1 - \gamma_1}{1 + \gamma_1} \geq 0. \end{aligned}$$

The last condition can be satisfied, because there are enough degrees of freedom in choosing the values of the multipliers in the RSW problem. In particular, it is satisfied for the dominant-strategy implementation, for which $\lambda_{11} = \lambda_{21} = 0$. ■

Proposition 5 implies that, whenever the null contract is RSW relative to itself, the principal can choose values for the multipliers to make the null contract the unique equilibrium at the collusion stage with fixed out-of-equilibrium beliefs following a rejection by agent 2. By doing so, the principal eliminates other equilibria at the collusion stage and thus, can ensure the second-best payoff. The next step is to analyze whether this is an optimal choice.

Proposition 6 *If the dominant-strategy implementation of the second-best contract is offered and the null contract is RSW relative to itself, the principal always strictly gains by making the null contract interim efficient relative to the prior beliefs.*

Proof. Suppose the second-best contract is weakly collusion-proof but is not strongly collusion-proof. That is, there is an equilibrium of the collusion game in which a non-null collusion contract is offered by A^1. Call this equilibrium $\overline{C} = \left\{\overline{\phi}_{k\ell}, \overline{y}_{k\ell}\right\}$. In this equilibrium at least one type of A^1 is strictly better off than with the null contract: for some k

$$\sum_{\ell} \nu_\ell \left(t^1\left(\overline{\phi}_{k\ell}\right) - \overline{y}_{k\ell} - \theta_k q\left(\overline{\phi}_{k\ell}\right)\right) > u_k^{1sb}.$$

According to Theorem I * in Maskin and Tirole (1992), any equilibrium of the collusion game satisfies the following conditions:

$$\begin{aligned}
\sum_{\ell} \nu_\ell \left(t^1\left(\overline{\phi}_{k\ell}\right) - \overline{y}_{k\ell} - \theta_k q\left(\overline{\phi}_{k\ell}\right)\right) &\geq \sum_{\ell} \nu_\ell \left(t^1\left(\overline{\phi}_{k'\ell}\right) - \overline{y}_{k'\ell} - \theta_k q\left(\overline{\phi}_{k'\ell}\right)\right) \quad \forall k, k', \\
\sum_{k} \nu_k \left(t^2\left(\overline{\phi}_{k\ell}\right) + \overline{y}_{k\ell} - \theta_\ell q\left(\overline{\phi}_{k\ell}\right)\right) &\geq \sum_{k} \nu_k \left(t^2\left(\overline{\phi}_{k\ell'}\right) + \overline{y}_{k\ell'} - \theta_\ell q\left(\overline{\phi}_{k\ell'}\right)\right) \quad \forall \ell, \ell', \\
\sum_{\ell} \nu_\ell \left(t^1\left(\overline{\phi}_{k\ell}\right) - \overline{y}_{k\ell} - \theta_k q\left(\overline{\phi}_{k\ell}\right)\right) &\geq u_k^{1sb} \quad \forall k, \\
\sum_{k} \nu_k \left(t^2\left(\overline{\phi}_{k\ell}\right) + \overline{y}_{k\ell} - \theta_\ell q\left(\overline{\phi}_{k\ell}\right)\right) &\geq u_\ell^{2sb} \quad \forall \ell, \qquad (6)
\end{aligned}$$

that is, the collusion contract is incentive compatible for the two agents, gives at least the same utility as the original (second-best) contract to agent 2 and gives more than the RSW payoff to agent 1. And because the second-best contract is weakly collusion-proof, the null contract is the RSW allocation and agent 1's RSW payoff is the second-best payoff.

If this collusion contract $\overline{C}$ is offered, the principal's payoff is

$$\sum_k \sum_\ell \nu_k \nu_\ell \left(S\left(q\left(\overline{\phi}_{k\ell}\right)\right) - t^1\left(\overline{\phi}_{k\ell}\right) - t^2\left(\overline{\phi}_{k\ell}\right)\right).$$

Define $\overline{q}_{k\ell} \equiv q\left(\overline{\phi}_{k\ell}\right)$, $\overline{t}^1_{k\ell} \equiv t^1\left(\overline{\phi}_{k\ell}\right)$, $\overline{t}^2_{k\ell} \equiv t^2\left(\overline{\phi}_{k\ell}\right)$ and suppose the principal offers the following contract: $m' = \left\{\overline{q}_{k\ell}, \overline{t}^1_{k\ell} - \overline{y}_{k\ell}, \overline{t}^2_{k\ell} + \overline{y}_{k\ell}\right\}$, so the null contract following this offer is an equilibrium of the collusion stage.

Given conditions (6), contract m' satisfies all the constraints of problem B. But instead, contract m^{sb} was chosen by the principal, meaning that

$$\sum_k \sum_\ell \nu_k \nu_\ell \left(S\left(q^{sb}_{k\ell}\right) - t^{1ds}_{k\ell} - t^{2ds}_{k\ell}\right) \geq \sum_k \sum_\ell \nu_k \nu_\ell \left(S\left(\overline{q}_{k\ell}\right) - \overline{t}^1_{k\ell} - \overline{t}^2_{k\ell}\right) \equiv \overline{W}.$$

$S(q)$ is a concave function so, $\forall k$, $\forall \ell$,[5]

$$S\left(\overline{q}_{k\ell}\right) \geq S\left(q\left(\overline{\phi}_{k\ell}\right)\right),$$

implying that

$$\overline{W} \geq \sum_k \sum_\ell \nu_k \nu_\ell \left(S\left(q\left(\overline{\phi}_{k\ell}\right)\right) - t^1\left(\overline{\phi}_{k\ell}\right) - t^2\left(\overline{\phi}_{k\ell}\right)\right).$$

Therefore, the principal is better off when there is no collusion. Moreover, the probability of collusion can be made equal to zero by choosing multipliers to make the null contract interim efficient relative to the prior beliefs. ■

The second-best contract maximizes the principal's payoff when there is no collusion. Collusion in this context can never be beneficial for the principal, because it would mean that lying is better than telling the truth for the principal, which contradicts the Revelation Principle. Therefore, if the collusion game has multiple equilibria, including the null contract, the principal's payoff when the null contract is selected is at least as high as in any other equilibrium. However, by choosing the values of the multipliers, the principal can guarantee that the unique equilibrium of the collusion game, when the dominant-strategy implementation of the second-best contract is offered, is the null contract. In this way, she is sure to obtain the second-best payoff, even when collusion is possible.

The result of this section implies that the coalition is not powerful enough to be able to change the outcome when the second-best contract is offered.

[5] Remember that $S\left(q\left(\overline{\phi}_{k\ell}\right)\right) = E_{\overline{\phi}_{k\ell}} S(q)$ and $S\left(\overline{q}_{k\ell}\right) = S\left(E_{\overline{\phi}_{k\ell}}(q)\right)$.

We could imagine that collusion would be more powerful if agent 1 could commit to punish agent 2 whenever the latter rejects the side contract. For instance, he could commit to announce a high-cost if the side contract is not accepted (assuming that he can indeed commit to such an announcement). This reduces agent 2's outside option because agent 2's rent decreases when agent 1 is inefficient and gives more power to agent 1 to implement collusion. Indeed, the dominant-strategy implementation of the second-best is not RSW relative to the new status quo of agent 2. A coalition of (θ_1, θ_2) would like to pretend to be (θ_2, θ_2). With this manipulation of reports agent 1 can impose a small penalty to agent 2 whenever the state is (θ_1, θ_1). When agent 1 could not commit to punish agent 2, this kind of deviation was impossible because a type 1 agent 2 would have rejected the penalty and would have chosen to play non-cooperatively the grand mechanism. If the punishment is credible, type 1 agent 2 accepts the penalty, because his utility is lower when agent 1 punishes by announcing a high-cost. Technically, what happens is that the multipliers of the participation constraints of a low-cost agent 2 in the RSW program, μ_{11} and μ_{21}, are now equal to 0. However, there is another implementation of the second-best contract that is RSW when $\mu_{21} = \mu_{11} = 0$. Of course, in order to avoid a manipulation of reports as mentioned before, the collusion constraint of a (θ_1, θ_2) coalition who pretends to be (θ_2, θ_2) has to be binding. Computing the multipliers for $\mu_{21} = \mu_{11} = 0$ and choosing $\gamma_1 < \nu_1$, this will give asymmetric transfers as follows:

$$\begin{aligned}
t_{11}^{1sb} &= \theta_1 q_{11}^{sb} + \nu_1 \Delta\theta q_{12}^{sb} + \nu_2 \Delta\theta q_{22}^{sb}, \\
t_{11}^{2sb} &= \theta_1 q_{11}^{sb} + \Delta\theta q_{21}^{sb}, \\
t_{12}^{1sb} &= \theta_1 q_{12}^{sb} + \nu_1 \Delta\theta q_{12}^{sb} + \nu_2 \Delta\theta q_{22}^{sb}, \\
t_{12}^{2sb} &= \theta_2 q_{12}^{sb}, \\
t_{21}^{1sb} &= \theta_2 q_{21}^{sb} + \nu_1 \Delta\theta \left(q_{21}^{sb} - q_{22}^{sb} \right), \\
t_{21}^{2sb} &= \theta_1 q_{21}^{sb} + \Delta\theta q_{22}^{sb}, \\
t_{22}^{1sb} &= \theta_2 q_{22}^{sb} - \frac{\nu_1^2}{\nu_2} \Delta\theta \left(q_{21}^{sb} - q_{22}^{sb} \right), \\
t_{22}^{2sb} &= \theta_2 q_{22}^{sb}.
\end{aligned}$$

We need condition $\gamma_1 < \nu_1$ in order to prevent a coalition of (θ_2, θ_1) from pretending to be (θ_2, θ_2) and to guarantee that the null contract is interim efficient relative to the prior beliefs (see Proposition 5). Thus, the same conclusions are drawn with a model in which collusion is more powerful in the sense that agent 1 can punish agent 2 following a rejection. Risk neutrality on the agents gives the principal enough degrees of freedom to find (asymmetric) transfers such that the second-best contract is weakly collusion-proof. The whole trick consists in rewarding (punishing) an inefficient agent 1 when he meets an efficient (inefficient) agent 2 and give a constant rent to an efficient agent 1 (independent of both agents' types). The second-best contract is still implemented in dominant strategies for agent 2. Because all the bargaining power at the collusion stage belongs to agent 1, there is no point in changing the implementation for agent 2. It is enough to offer a contract such that agent 1 cannot improve by offering a collusion contract. Nevertheless, notice that agent 1 gets a negative utility when the state of nature is (θ_2, θ_2) in order to discourage him from proposing such a manipulation of reports.

4 Ex ante collusion

We have shown in the previous section that collusion is not an issue if types are independent. This result depends, however, on a strong assumption about communication. It is assumed that the agents accept the principal's offer before collusion takes place. Implicitly, this means that, before collusion, there is communication between the principal and the agents, but this communication is limited. Indeed, if the principal can communicate with the agents before the agents communicate with each other, what prevents her from forcing the agents to send messages about their types at the same time? This would prevent any possible collusion. A more plausible assumption is that collusion between the agents happens before accepting or rejecting the principal's offer. This is typically the case in an auction, in which participation takes place with some delay after the design, giving time to the potential participants to communicate with each other before deciding whether to participate or not. Following Dequiedt (2002), we assume in this section that agents collude before accepting or rejecting the principal's offer. The effect of such an assumption is that the participation constraints have to be satisfied type by type, because the type of agent i is revealed to agent j at the collusion stage. Therefore, when the agents accept the principal's offer, they know each other's type. We know from the results of the previous section that the principal can still implement the second-best contract with type by type participation constraints. She just needs to offer the dominant-strategy implementation. Thus, in order to make the problem interesting, we also assume that the agent who offers the collusion contract can punish the other agent whenever the latter rejects the side contract.

Punishing agent 2 may be an ex post suboptimal strategy for agent 1. However, agent 1 can find some means to commit to play an ex post suboptimal strategy. Consider, for instance, the following mechanism offered by agent 1. Following the principal's offer, agent 1 offers a contract to agent

2 and to an uninformed witness. This contract consists of the side contract between agent 1 and agent 2 and specifies the punishment strategy if agent 2 rejects the offer and a small payment ε to the witness if he accepts the offer. Moreover, agent 1's offer includes a bank deposit equal to π, which goes back to agent 1 if agent 2 accepts the offer. The contract specifies that whenever agent 2 rejects the offer, if agent 1 follows the punishment strategy, he can get the deposit back. If agent 1 deviates from the punishment strategy, the witness can enforce the contract and get π for himself. If agent 1 deviates from the punishment strategy and the witness does not enforce the contract, the money goes to charity. The penalty π can always be made large enough in order to force agent 1 to implement the punishment strategy whatever his type. The witness always accepts the offer because he is sure to get ε, and, therefore the punishment strategy becomes enforceable even though it is not *a priori* ex post optimal. Moreover, with this mechanism there is no scope for renegotiation between agent 1 and the witness.

The new timing is as follows:

1. P offers a mechanism, m^P, to A^1 and A^2. m^P determines a quantity to be produced and a vector of transfers for the two agents as functions of their reports: $\left\{q\left(\widetilde{\theta}^1,\widetilde{\theta}^2\right),t^1\left(\widetilde{\theta}^1,\widetilde{\theta}^2\right),t^2\left(\widetilde{\theta}^1,\widetilde{\theta}^2\right)\right\}$ where $\widetilde{\theta}^k$ is A^k's announcement of his own marginal cost in m^P.

2. A^1 offers a collusion contract, C, to A^2. C determines a manipulation of reports (ϕ), a transfer (y) from A^1 to A^2 as functions of both agent's reports: $\left\{\phi\left(\widehat{\theta}^1,\widehat{\theta}^2\right),y\left(\widehat{\theta}^1,\widehat{\theta}^2\right)\right\}$, where $\widehat{\theta}^k$ is A^k's announcement of his own marginal cost in C and a punishment function $(x(\theta_1))$ if A^2 rejects A^1's offer. The punishment strategy is the announcement of A^1 in m^P if A^2 rejects the collusion contract.

3. A^2 accepts or rejects A^1's offer. If A^2 rejects the offer, A^1 and A^2 go to stage 5. If A^2 accepts A^1's offer, they go to stage 4.

4. A^1 and A^2 simultaneously announce their types to each other.

5. A^1 and A^2 simultaneously accept or reject P's offer. If one of them rejects, the game is over and everyone obtains 0. If they both accept, they go to stage 6.

6. A^1 and A^2 simultaneously announce their types to the principal (according to the function ϕ if they agreed on a collusion contract or according to the function x for A^1 if they did not).

7. Production is carried-out and transfers are paid.

First, notice that the second-best contract without collusion is not affected by this change in the timing, because if there is no collusion, the agents have to accept or reject the principal's offer under asymmetric information, so participation constraints are still interim.[6] Second, the status quo utility at the collusion stage, given by the payoff of A^2 if he rejects the collusion offer, is determined by the punishment strategy of A^1. Moreover, if the punishment strategy is independent of A^1's type, the common value component of the collusion game disappears and the problem can be analyzed under the private value framework. We will show that it is indeed optimal for agent 1 to commit to set his punishment strategy independent of his type.

Proposition 7 *It is optimal for Agent 1 to commit himself to set a punishment strategy independent of his type.*

Proof. See Appendix. ■

By committing to a type independent punishment strategy, agent 1 transforms the collusion game in an informed principal problem with private values. A property of the private value case is that the equilibrium is unique. On the contrary, if the punishment strategy depends on agent 1's type, the game remains a common value problem, characterized, in general, by a multiplicity of equilibria. Among all equilibria, each type of agent 1 finds one particular equilibrium that gives him the highest payoff. The idea is that agent 1, whatever his type, can reproduce his best possible equilibrium in a common value context with a type independent punishment strategy. Moreover, he avoids the problem of multiplicity of equilibria inherent to the common value framework, eliminating all equilibria that give him a lower payoff. He can do at least as well with a type independent punishment strategy as in the best possible equilibrium with type dependent punishment strategy. Therefore, he does strictly better than in any other equilibrium.

4.1 Type independent punishment strategy

Suppose that agent 1 sets a punishment strategy independent of his private information:

$$x(\theta_1) = x(\theta_2) = x \in \{\theta_1, \theta_2\}.$$

[6] Even if participation constraints were ex post, the second best contract without collusion would be the same, because, as we showed before, the second-best contract can be implemented in dominant strategies.

Then, the utility of a type j agent 2 if he rejects the collusion contract is given by

$$u_0^2(\theta_j /x) = \max \left\{ 0, \max_{\widetilde{\theta}\in\{\theta_1,\theta_2\}} \left[t^2\left(x,\widetilde{\theta}\right) - \theta_j q\left(x,\widetilde{\theta}\right)\right]\right\},$$

that is, agent 2 chooses the best strategy in the gran contract, given the punishment strategy of agent 1. Agent 2 has always the possibility of rejecting the grand contract and, then, can always guarantee himself a 0 payoff. Because the punishment strategy is independent of agent 1's type, the problem has to be analyzed in the private value context. Using the results in Maskin and Tirole (1990) and Quesada (2002) we can characterize the solution of the collusion game.

Proposition 8 *a) If the punishment strategy is $x = \theta_2$ then a grand contract is collusion-proof if and only if it satisfies the following conditions:*

$$\begin{aligned} t_{11}^2 - \theta_1 q_{11} &= t_{12}^2 - \theta_1 q_{12}, \\ t_{21}^2 - \theta_1 q_{21} &= t_{22}^2 - \theta_1 q_{22} \geq \max\left\{0, u_0^2(\theta_1 /\theta_2)\right\}, \\ t_{12}^2 - \theta_2 q_{12} &= \max\left\{0, u_0^2(\theta_2 /\theta_2)\right\}, \\ t_{22}^2 - \theta_2 q_{22} &= \max\left\{0, u_0^2(\theta_2 /\theta_2)\right\}, \end{aligned} \tag{7}$$

$$t_{ij}^1 - \theta_i q_{ij} \geq 0 \quad \forall i, \forall j, \tag{8}$$

and the coalition constraints: $\forall i, \forall j$

$$\begin{aligned} t_{11}^1 + t_{11}^2 - 2\theta_1 q_{11} &\geq t_{ij}^1 + t_{ij}^2 - 2\theta_1 q_{ij}, \\ t_{21}^1 + t_{21}^2 - (\theta_1+\theta_2) q_{21} &\geq t_{ij}^1 + t_{ij}^2 - (\theta_1+\theta_2) q_{ij}, \\ t_{12}^1 + t_{12}^2 - (\theta_1+\theta_2) q_{12} - \frac{\nu_1}{\nu_2}\Delta\theta q_{12} &\geq t_{ij}^1 + t_{ij}^2 - (\theta_1+\theta_2) q_{ij} - \frac{\nu_1}{\nu_2}\Delta\theta q_{ij}, \\ t_{22}^1 + t_{22}^2 - 2\theta_2 q_{22} - \frac{\nu_1}{\nu_2}\Delta\theta q_{22} &\geq t_{ij}^1 + t_{ij}^2 - 2\theta_2 q_{ij} - \frac{\nu_1}{\nu_2}\Delta\theta q_{ij}. \end{aligned} \tag{9}$$

b) Given any collusion-proof contract, agent 1 weakly prefers to set his punishment strategy at $x = \theta_2$.

We first prove the following lemma on monotonicity of the quantity profile, and we prove then Proposition 8.

Lemma 1 *If the punishment strategy is type-independent, any collusion-proof mechanism satisfies the monotonicity condition $q_{11} \geq q_{21} \geq q_{12} \geq q_{22}$.*

Proof. A grand contract $\left(t_{ij}^1, t_{ij}^2, q_{ij}\right)$ is collusion-proof if the null contract is a solution of the collusion game. Given that the collusion game fits the private value framework, and because agents are risk neutral, we can analyze this game as if agent 2 knew agent 1's marginal cost.

If agent 1 is type i, the best contract he could offer to agent 2 is the contract that solves

$$
\begin{array}{ll}
\max\limits_{\left(\phi_j, y_j\right)_{j=1,2}} \sum\limits_j \nu_j \left[t^1\left(\phi_{ij}\right) - \theta_i q\left(\phi_{ij}\right) - y_{ij}\right] & \\
\text{subject to} & \\
t^2\left(\phi_{i1}\right) - \theta_1 q\left(\phi_{i1}\right) + y_{i1} \geq t^2\left(\phi_{i2}\right) - \theta_1 q\left(\phi_{i2}\right) + y_{i2} & \left(\lambda_{i1}\right) \\
t^2\left(\phi_{i2}\right) - \theta_2 q\left(\phi_{i2}\right) + y_{i2} \geq t^2\left(\phi_{i1}\right) - \theta_2 q\left(\phi_{i1}\right) + y_{i1} & \left(\lambda_{i2}\right) \\
t^2\left(\phi_{i1}\right) - \theta_1 q\left(\phi_{i1}\right) + y_{i1} \geq \max\left\{0, u_0^2\left(\theta_1 / x\right)\right\} & \left(\mu_{i1}\right) \\
t^2\left(\phi_{i2}\right) - \theta_2 q\left(\phi_{i2}\right) + y_{i2} \geq \max\left\{0, u_0^2\left(\theta_2 / x\right)\right\} & \left(\mu_{i2}\right)
\end{array}
$$

If the null contract is a solution to this problem, then the grand contract satisfies the following conditions:

$$
\begin{aligned}
t_{11}^2 - \theta_1 q_{11} &\geq t_{12}^2 - \theta_1 q_{12}, && (10) \\
t_{12}^2 - \theta_2 q_{12} &\geq t_{11}^2 - \theta_2 q_{11}, && (11) \\
t_{21}^2 - \theta_1 q_{21} &\geq t_{22}^2 - \theta_1 q_{22}, && (12) \\
t_{22}^2 - \theta_2 q_{22} &\geq t_{21}^2 - \theta_2 q_{21}, && (13) \\
t_{11}^2 - \theta_1 q_{11} &\geq \max\left\{0, u_0^2\left(\theta_1 / x\right)\right\}, && (14) \\
t_{12}^2 - \theta_2 q_{12} &\geq \max\left\{0, u_0^2\left(\theta_2 / x\right)\right\}, && (15) \\
t_{21}^2 - \theta_1 q_{21} &\geq \max\left\{0, u_0^2\left(\theta_1 / x\right)\right\}, && (16) \\
t_{22}^2 - \theta_2 q_{22} &\geq \max\left\{0, u_0^2\left(\theta_2 / x\right)\right\}. && (17)
\end{aligned}
$$

and the incentive compatibility constraints of agent 1 (with private values, the incentive constraints of the party who offers the contract are never binding). Then, from (10) and (11), and from (12) and (13) we obtain

$$
\begin{aligned}
q_{11} &\geq q_{12}, \\
q_{21} &\geq q_{22}. && (18)
\end{aligned}
$$

Moreover, given conditions (10) to (13), we have that

$$
\begin{aligned}
u_0^2\left(\theta_1 / x\right) &= t^2\left(x, \theta_1\right) - \theta_1 q\left(x, \theta_1\right) \geq 0, && (19) \\
u_0^2\left(\theta_2 / x\right) &= t^2\left(x, \theta_2\right) - \theta_2 q\left(x, \theta_2\right) \geq 0. && (20)
\end{aligned}
$$

Optimizing with respect to y_{ij} we obtain the first order conditions:

$$\begin{aligned}
\lambda_{11} - \lambda_{12} + \mu_{11} &= \nu_1, \\
\lambda_{12} - \lambda_{11} + \mu_{12} &= \nu_2, \\
\lambda_{21} - \lambda_{22} + \mu_{21} &= \nu_1, \\
\lambda_{22} - \lambda_{21} + \mu_{22} &= \nu_2.
\end{aligned}$$

Optimizing with respect to ϕ_{ij} and using the conditions above we obtain:

$$\begin{aligned}
(\theta_1, \theta_1) &\in \arg\max_{\widetilde{\phi}} \left\{ t^1\left(\widetilde{\phi}\right) + t^2\left(\widetilde{\phi}\right) - 2\theta_1 q\left(\widetilde{\phi}\right) + \frac{\lambda_{12}}{\nu_1}\Delta\theta q\left(\widetilde{\phi}\right) \right\}, \\
(\theta_1, \theta_2) &\in \arg\max_{\widetilde{\phi}} \left\{ t^1\left(\widetilde{\phi}\right) + t^2\left(\widetilde{\phi}\right) - (\theta_1 + \theta_2)\, q\left(\widetilde{\phi}\right) - \frac{\lambda_{11}}{\nu_2}\Delta\theta q\left(\widetilde{\phi}\right) \right\}, \\
(\theta_2, \theta_1) &\in \arg\max_{\widetilde{\phi}} \left\{ t^1\left(\widetilde{\phi}\right) + t^2\left(\widetilde{\phi}\right) - (\theta_1 + \theta_2)\, q\left(\widetilde{\phi}\right) + \frac{\lambda_{22}}{\nu_1}\Delta\theta q\left(\widetilde{\phi}\right) \right\}, \\
(\theta_2, \theta_2) &\in \arg\max_{\widetilde{\phi}} \left\{ t^1\left(\widetilde{\phi}\right) + t^2\left(\widetilde{\phi}\right) - 2\theta_2 q\left(\widetilde{\phi}\right) - \frac{\lambda_{21}}{\nu_2}\Delta\theta q\left(\widetilde{\phi}\right) \right\}. \qquad (21)
\end{aligned}$$

Case 1: the punishment strategy is $x = \theta_2$.

Suppose that at the collusion stage the participation constraint of the high-cost agent 2 and the incentive constraints of the low-cost agent 2 are binding (we will verify ex post that the other constraints are satisfied too). Then, transfers are given by

$$\begin{aligned}
t^2_{11} &= \theta_1 q_{11} + \Delta\theta q_{12} + u^2_0\left(\theta_2 / \theta_2\right), \\
t^2_{12} &= \theta_2 q_{12} + u^2_0\left(\theta_2 / \theta_2\right), \\
t^2_{21} &= \theta_1 q_{21} + \Delta\theta q_{22} + u^2_0\left(\theta_2 / \theta_2\right), \\
t^2_{22} &= \theta_2 q_{22} + u^2_0\left(\theta_2 / \theta_2\right),
\end{aligned}$$

and multipliers are $\lambda_{12} = \lambda_{22} = \mu_{11} = \mu_{21} = 0$, $\lambda_{11} = \lambda_{21} = \nu_1$, $\mu_{12} = \mu_{22} = 1$.

Replacing in (21), we have that telling the truth is an optimal strategy for the coalition if

$$\begin{aligned}
(\theta_1, \theta_1) &\in \arg\max_{\widetilde{\phi}} \left\{ t^1\left(\widetilde{\phi}\right) + t^2\left(\widetilde{\phi}\right) - 2\theta_1 q\left(\widetilde{\phi}\right) \right\}, \\
(\theta_1, \theta_2) &\in \arg\max_{\widetilde{\phi}} \left\{ t^1\left(\widetilde{\phi}\right) + t^2\left(\widetilde{\phi}\right) - (\theta_1 + \theta_2)\, q\left(\widetilde{\phi}\right) - \frac{\nu_1}{\nu_2}\Delta\theta q\left(\widetilde{\phi}\right) \right\}, \\
(\theta_2, \theta_1) &\in \arg\max_{\widetilde{\phi}} \left\{ t^1\left(\widetilde{\phi}\right) + t^2\left(\widetilde{\phi}\right) - (\theta_1 + \theta_2)\, q\left(\widetilde{\phi}\right) \right\}, \\
(\theta_2, \theta_2) &\in \arg\max_{\widetilde{\phi}} \left\{ t^1\left(\widetilde{\phi}\right) + t^2\left(\widetilde{\phi}\right) - 2\theta_2 q\left(\widetilde{\phi}\right) - \frac{\nu_1}{\nu_2}\Delta\theta q\left(\widetilde{\phi}\right) \right\}.
\end{aligned}$$

Moreover, these conditions imply the monotonicity constraints

$$q_{11} \geq q_{21} \geq q_{12} \geq q_{22}.$$

We verify now that the other constraints are satisfied. For that to be the case we need

$$\begin{aligned} u_0^2(\theta_1/\theta_2) - u_0^2(\theta_2/\theta_2) &\leq \Delta\theta q_{12}, \\ u_0^2(\theta_1/\theta_2) - u_0^2(\theta_2/\theta_2) &\leq \Delta\theta q_{22}, \\ q_{11} &\geq q_{12}, \\ q_{21} &\geq q_{22}. \end{aligned}$$

The last two constraints come immediately from (18). According to (19) and (20), we have that

$$\begin{aligned} u_0^2(\theta_1/\theta_2) &= t_{21}^2 - \theta_1 q_{21} = \overline{u}_{22}^2 + \Delta\theta q_{22}, \\ u_0^2(\theta_2/\theta_2) &= t_{22}^2 - \theta_2 q_{22} = \overline{u}_{22}^2, \end{aligned}$$

and therefore

$$u_0^2(\theta_1/\theta_2) - u_0^2(\theta_2/\theta_2) = \Delta\theta q_{22} \leq \Delta\theta q_{12}.$$

Case 2: the punishment strategy is $x = \theta_1$.

In this case, the previous solution does not satisfy the participation constraint of a low-cost agent 2 when agent 1 is high-cost if $q_{12} > q_{22}$. So this constraint has to be binding at the collusion stage: $\mu_{21} > 0$. Suppose that the multipliers are $\lambda_{12} = \lambda_{22} = \mu_{11} = 0$, $\lambda_{11} = \nu_1$, $\mu_{12} = 1$, $\mu_{21} = \nu_1 - \lambda_{21}$, $\mu_{22} = \nu_2 + \lambda_{21}$ and $\nu_1 \geq \lambda_{21} \geq \max\{0, \nu_1 - \nu_2\}$ and $\lambda_{21} > 0 \Rightarrow q_{12} = q_{22}$. This implies also that $(q_{12} - q_{22})(\nu_2 - \nu_1) \geq 0$. We will check that indeed the other constraints are satisfied at the optimal solution.

Replacing in the objective function, we have that the optimal manipulation of reports function is given by

$$\begin{aligned} (\theta_1, \theta_1) &\in \arg\max_{\widetilde{\phi}} \left\{ t^1\left(\widetilde{\phi}\right) + t^2\left(\widetilde{\phi}\right) - 2\theta_1 q\left(\widetilde{\phi}\right) \right\}, \\ (\theta_1, \theta_2) &\in \arg\max_{\widetilde{\phi}} \left\{ t^1\left(\widetilde{\phi}\right) + t^2\left(\widetilde{\phi}\right) - (\theta_1 + \theta_2) q\left(\widetilde{\phi}\right) - \frac{\nu_1}{\nu_2} \Delta\theta q\left(\widetilde{\phi}\right) \right\}, \\ (\theta_2, \theta_1) &\in \arg\max_{\widetilde{\phi}} \left\{ t^1\left(\widetilde{\phi}\right) + t^2\left(\widetilde{\phi}\right) - (\theta_1 + \theta_2) q\left(\widetilde{\phi}\right) \right\}, \\ (\theta_2, \theta_2) &\in \arg\max_{\widetilde{\phi}} \left\{ t^1\left(\widetilde{\phi}\right) + t^2\left(\widetilde{\phi}\right) - 2\theta_2 q\left(\widetilde{\phi}\right) - \frac{\lambda_{21}}{\nu_2} \Delta\theta q\left(\widetilde{\phi}\right) \right\}. \end{aligned}$$

Moreover, given that $\lambda_{21} \geq \nu_1 - \nu_2$, these conditions imply the monotonicity constraints

$$q_{11} \geq q_{21} \geq q_{12} \geq q_{22},$$

We verify now that the other constraints are satisfied. For that to be the case we need

$$\begin{array}{rcl} u_0^2(\theta_1/\theta_1) - u_0^2(\theta_2/\theta_1) & \leq & \Delta\theta q_{12}, \\ u_0^2(\theta_1/\theta_1) - u_0^2(\theta_2/\theta_1) & \geq & \Delta\theta q_{22}, \\ q_{11} & \geq & q_{12}, \\ q_{21} & \geq & q_{22}. \end{array}$$

The last two constraints come immediately from (18). According to (19) and (20), we have that

$$\begin{array}{rcl} u_0^2(\theta_1/\theta_1) & = & t_{11}^2 - \theta_1 q_{11} = \overline{u}_{12}^2 + \Delta\theta q_{12}, \\ u_0^2(\theta_2/\theta_1) & = & t_{12}^2 - \theta_2 q_{12} = \overline{u}_{12}^2, \end{array}$$

and therefore

$$u_0^2(\theta_1/\theta_2) - u_0^2(\theta_2/\theta_2) = \Delta\theta q_{12} \geq \Delta\theta q_{22}.$$

■

Proof of Proposition 8. a) The proof follows immediately from the proof of Lemma 1, case 1 and from the fact that the participation constraints of agent 1 have to be satisfied.

b) Following the proof of Lemma 1, if the punishment strategy is $x = \theta_1$, a collusion-proof grand contract has to satisfy:

$$\begin{array}{rcl} t_{11}^2 - \theta_1 q_{11} & = & t_{12}^2 - \theta_1 q_{12}, \\ t_{21}^2 - \theta_1 q_{21} & = & u_0^2(\theta_1/\theta_1) \geq t_{22}^2 - \theta_1 q_{22}, \\ t_{12}^2 - \theta_2 q_{12} & = & u_0^2(\theta_2/\theta_1), \\ t_{22}^2 - \theta_2 q_{22} & = & u_0^2(\theta_2/\theta_1). \end{array} \tag{22}$$

Given that the contract has to satisfy the type by type incentive and participation constraints for agent 2, we have that, for any punishment stratgegy x,

$$u_0^2(\theta_j/x) = t_{xj}^2 - \theta_j q_{xj}.$$

Take any collusion-proof contract with some $u_{22}^2 = t_{22}^2 - \theta_2 q_{22}$. Using (7) and (22) we can determine all the other transfers to agent 2, for a given punishment strategy:

$$\begin{array}{ll} \qquad x = \theta_2 & \qquad x = \theta_1 \\ t_{11}^2 = \theta_1 q_{11} + \Delta\theta q_{12} + u_{22}^2 & t_{11}^2 = \theta_1 q_{11} + \Delta\theta q_{12} + u_{22}^2 \\ t_{21}^2 = \theta_1 q_{21} + \Delta\theta q_{22} + u_{22}^2 & t_{21}^2 = \theta_1 q_{21} + \Delta\theta q_{12} + u_{22}^2 \\ t_{12}^2 = \theta_2 q_{12} + u_{22}^2 & t_{12}^2 = \theta_2 q_{12} + u_{22}^2 \end{array}$$

and $q_{12} \geq q_{22}$ because the contract is collusion-proof.

It follows that

$$\begin{aligned} u_0^2\left(\theta_2 /\theta_1\right) &= u_0^2\left(\theta_2 /\theta_2\right) = u_{22}^2, \\ u_0^2\left(\theta_1 /\theta_1\right) &= u_{22}^2 + \Delta\theta q_{12} \geq u_0^2\left(\theta_1 /\theta_2\right) = u_{22}^2 + \Delta\theta q_{22}. \end{aligned}$$

The outside option of agent 2 at the collusion stage is weakly smaller when $x = \theta_2$. Agent 1's payoff is decreasing in agent 2's outside option, therefore, agent 1 weakly prefers $x = \theta_2$. ■

We have found necessary and sufficient conditions for a grand contract to be collusion-proof, provided that agent 1's punishment strategy is independent of his own type. From an ex ante point of view (before knowing agent 2's type), the optimal punishment strategy is to announce θ_2 whenever agent 2 rejects the side contract. The intuition is that by announcing θ_1 agent 1 increases the outside option of agent 2, given that any contract offered by the principal has to satisfy the ex post incentive constraints for agent 2 in order to be collusion-proof. Increasing agent 2's outside option can only decrease agent 1's payoff. Therefore, agent 1 can obtain more from the collusion if he threatens agent 2 with a harsh announcement in case of rejection. Similarly, it is weakly preferred to set $x = \theta_2$ to any randomization between θ_1 and θ_2. There is another possible punishment that we did not consider here. Agent 1 could punish agent 2 by rejecting the principal's offer. However, we can easily see that agent 1 cannot do better with such a punishment. Threatening agent 2 with a rejection of the grand mechanism certainly decreases the outside option of a low-cost agent 2, who gets a strictly positive utility if the contract is signed in any collusion-proof contract. Nevertheless, the participation constraints of a low-cost agent 2 are non-binding in the optimal collusion contract and, thus, the decrease in the low-cost agent 2's outside option does not increase agent 1's payoff. Therefore, the only benefit would come from a decrease of the outside option of a high-cost agent 2. But the principal wants to minimize agent 2's utility and, thus, any optimal collusion-proof contract will have the inefficient agent 2's utility equal to 0 and agent 1 cannot decrease it further.

Collusion is efficient (i.e., maximizes the sum of the agents' utilities) whenever agent 2 is efficient. As usual, there is no gain from introducing inefficiencies in side contracting when agent 2 is efficient. However, a distortion in side contracting appears whenever agent 2 is inefficient. This distortion helps reducing the informational rent that agent 1 has to give up in order to obtain truthful revelation from a low-cost agent 2. When agent 2's production cost is high, the coalition does not maximize joint utility, but a "virtual" joint utility, in which the virtual (aggregate) cost is higher than the total production cost because it includes the (expected) cost of the informational rent from 1 to 2. The difference between the virtual cost and the production cost is a measure of the frictions in side contracting created by asymmetric information. The private value feature of the model and the assumption of risk neutrality imply that the distortion is the same that would arise in a model in which agent 1's private information were known by agent 2. At this point, it is important to bring attention to the problem of multiple equilibria at the side contracting stage. It is shown in Quesada (2002) that when both parties are risk neutral, the informed principal problem has an infinity of equilibria. Nevertheless, agent 1's (the principal in the side contract) payoff, whatever his type, is the same in all equilibria. This property allows us to neglect the problem of equilibrium selection without loss of generality. Indeed, suppose that, given a grand contract m^P, there is one equilibrium in which agent 1 strictly gains by offering a non-null collusion contract. Then, the null contract is never an equilibrium collusion contract, because agent 1 would also gain by offering any other side contract that would be an equilibrium of the collusion game following m^P. Similarly, if the null contract is one equilibrium of the collusion game given m^P, no other equilibrium can do strictly better than the null contract and, therefore, m^P is strongly collusion-proof.

Another interesting point is that the incentive constraints for agent 1 are always satisfied, even though they have been neglected in the analysis. This is also due to the private value framework. When agent 1 is efficient his incentive constraint is just satisfied, while the incentive constraint of an inefficient agent 1 is slack.

With all these ingredients we are ready to show that collusion is now powerful enough to improve with respect to the second-best contract.

Proposition 9 *The second-best contract is not collusion-proof.*

Proof. First, notice that for the second-best contract to be accepted at stage 5 it has to satisfy the ex post participation constraints for both agents.

Then, we have

$$\begin{aligned} t_{21}^{1sb} &= t_{12}^{2sb} = \theta_2 q_{12}^{sb}, \\ t_{22}^{1sb} &= t_{22}^{2sb} = \theta_2 q_{22}^{sb}, \end{aligned}$$

because $q_{21}^{sb} = q_{12}^{sb}$.

Conditions (7) imply also that

$$\begin{aligned} t_{11}^{2sb} &= \theta_1 q_{11}^{sb} + \Delta\theta q_{12}^{sb}, \\ t_{21}^{2sb} &= \theta_1 q_{12}^{sb} + \Delta\theta q_{22}^{sb}. \end{aligned}$$

From (9) we have that

$$\begin{aligned} & t_{12}^{1sb} + t_{12}^{2sb} - (\theta_1 + \theta_2) q_{12}^{sb} - \frac{\nu_1}{\nu_2} \Delta\theta q_{12}^{sb} \\ \geq \; & t_{21}^{1sb} + t_{21}^{2sb} - (\theta_1 + \theta_2) q_{12}^{sb} - \frac{\nu_1}{\nu_2} \Delta\theta q_{12}^{sb}, \end{aligned}$$

and

$$\begin{aligned} & t_{21}^{1sb} + t_{21}^{2sb} - (\theta_1 + \theta_2) q_{12}^{sb} \\ \geq \; & t_{12}^{1sb} + t_{12}^{2sb} - (\theta_1 + \theta_2) q_{12}^{sb}, \end{aligned}$$

meaning that

$$t_{12}^{1sb} + t_{12}^{2sb} = t_{21}^{1sb} + t_{21}^{2sb},$$

or

$$t_{12}^{1sb} = \theta_1 q_{12}^{sb} + \Delta\theta q_{22}^{sb}.$$

Finally, the second-best contract satisfies with equality the interim incentive constraint of an efficient agent 1:

$$\nu_1 \left(t_{11}^{1sb} - \theta_1 q_{11}^{sb} \right) + \nu_2 \left(t_{12}^{1sb} - \theta_1 q_{12}^{sb} \right) = \Delta\theta \left(\nu_1 q_{12}^{sb} + \nu_2 q_{22}^{sb} \right),$$

so

$$t_{11}^{1sb} = \theta_1 q_{11}^{sb} + \Delta\theta q_{12}^{sb}.$$

So, actually, the second-best contract has to be implemented in dominant strategies. Now, agent 1 has incentives to offer a collusion contract in which in state (θ_1, θ_2) the coalition announces (θ_2, θ_2), and punish agent 2 by announcing θ_2 if he rejects. Indeed, he does not need to compensate a high-cost agent 2 for this change of announcement because the latter will get 0 anyway. On the contrary, he can impose a penalty to agent 2 in state (θ_1, θ_1), $y_{11} = -\Delta\theta \left(q_{12}^{sb} - q_{22}^{sb}\right) < 0$. ■

When agents collude before accepting the principal's offer, agent 1 can always find a collusion contract different from the null contract that makes him better off if the principal offers the second-best contract. Therefore, the second-best contract is not collusion-proof and the principal has to offer another grand contract among the set of collusion-proof contracts. The difference with the case in which agents accept or reject the grand contract before colluding is that the principal loses some degrees of freedom that have to be used to satisfy the ex post participation constraints instead of the collusion constraints. Moreover, knowing that collusion proofness requires that agent 2's incentive constraints be satisfied type by type, another degree of freedom is lost in this operation. The second-best quantity and the aggregate marginal cost are the same in states (θ_1, θ_2) and (θ_2, θ_1), then, to prevent collusion the principal has to give the same total transfer in both states of nature. Thus, the last degree of freedom is exhausted. Therefore, if there is a way to implement the second-best contract it has to be in dominant strategies. But then, agent 1 has incentives to collectively announce (θ_2, θ_2) when the true state is (θ_1, θ_2). By doing so, an efficient agent 1 decreases the rent he has to give to an efficient agent 2 to obtain truthful revelation. Indeed, suppose both agents are efficient: the true state of the world is (θ_1, θ_1). Agent 1 wants agent 2 to reveal his true type at the collusion stage, so he has to guarantee a utility at least equal to what agent 2 would get by lying, which depends on the announcement of a coalition (θ_1, θ_2):

$$t^2 \left(\phi_{12}\right) + y_{12} - \theta_1 q \left(\phi_{12}\right). \tag{23}$$

Suppose $y_{12} = 0$. If a coalition (θ_1, θ_2) is truthful, (23) becomes

$$t_{12}^{2ds} - \theta_1 q_{12}^{sb} = \Delta\theta q_{12}^{sb}.$$

If instead, a coalition (θ_1, θ_2) announces (θ_2, θ_2), (23) becomes

$$t_{22}^{2ds} - \theta_1 q_{22}^{sb} = \Delta\theta q_{22}^{sb} < \Delta\theta q_{12}^{sb}.$$

Therefore, agent 1 can reduce the utility of agent 2 in state (θ_1, θ_1) by imposing a fee on agent 2 equal to $\Delta\theta \left(q_{12}^{sb} - q_{22}^{sb}\right)$, the stakes for collusion. Thus, a low-cost agent 1 is strictly better off by offering this collusion contract.

It is the interaction of both the ex post participation constraints and the punishment strategy what makes the second-best contract non-implementable when collusion is possible. Indeed, we have proven in the previous section that only one of those conditions is not enough to prevent the principal from implementing the second-best contract. On the one hand, if agent 1 does not have access to a punishment technology, the principal can satisfy ex post participation constraints by increasing the status quo utility level of agent 2 in order to make any collusion offer very expensive for agent 1. On the other hand, if a punishment technology is available but participation occurs in an interim stage, the principal can still implement the second-best contract by offering a contract that gives a negative utility to agent 1 whenever both agents are inefficient, destroying the stake of collusion. However, when agent 1 has access to a punishment technology and participation is ex post, the principal cannot manage anymore to implement the second-best contract. Agent 1 can always take advantage of the second-best contract with a side contract in which a coalition of (θ_1, θ_2) announces (θ_2, θ_2). By doing so, he relaxes the incentive constraint of a low-cost agent 2, because $q_{22}^{sb} < q_{12}^{sb}$, and, therefore, can reduce the transfer (ask for a fee) to a low-cost agent 2. Finally, agent 2 is willing to accept to pay such a fee because agent 1 threatens to punish him by announcing a high cost in the grand mechanism.

The second-best contract, then, cannot be an optimal contract for the principal because it cannot be implemented when collusion is possible. In Proposition 8 we have obtained the characteristics of the set of collusion-proof (implementable) contracts. We now look at the best offer of the principal.

Proposition 10 *The best collusion-proof contract is characterized by the following conditions on transfers:*

$$
\begin{aligned}
t_{11}^{1c} &= \theta_1 q_{11}^c + \Delta\theta \left(q_{21}^c - q_{12}^c + q_{22}^c\right) + \frac{\nu_1}{\nu_2}\Delta\theta \left(q_{12}^c - q_{22}^c\right), \\
t_{12}^{1c} &= \theta_1 q_{12}^c + \Delta\theta q_{22}^c + \frac{\nu_1}{\nu_2}\Delta\theta \left(q_{12}^c - q_{22}^c\right), \\
t_{21}^{1c} &= \theta_2 q_{21}^c + \frac{\nu_1}{\nu_2}\Delta\theta \left(q_{12}^c - q_{22}^c\right), \\
t_{22}^{1c} &= \theta_2 q_{22}^c;
\end{aligned}
$$

$$
\begin{aligned}
t_{11}^{2c} &= \theta_1 q_{11}^c + \Delta\theta q_{12}^c, \\
t_{12}^{2c} &= \theta_2 q_{12}^c, \\
t_{21}^{2c} &= \theta_1 q_{21}^c + \Delta\theta q_{22}^c, \\
t_{22}^{2c} &= \theta_2 q_{22}^c.
\end{aligned}
$$

Optimal quantities are given by:

1. if $\nu_2^2 \geq \nu_1$,

$$\begin{aligned}
S'\left(q_{11}^c\right) &= 2\theta_1, \\
S'\left(q_{21}^c\right) &= \theta_1 + \theta_2 + \frac{\nu_1}{\nu_2}\Delta\theta, \\
S'\left(q_{12}^c\right) &= \theta_1 + \theta_2 + \frac{1-\nu_2^2}{\nu_2^2}\Delta\theta, \\
S'\left(q_{22}^c\right) &= 2\theta_2 + \frac{1-\nu_2^2}{\nu_2^2}\frac{\nu_2-\nu_1}{\nu_2}\Delta\theta; \qquad (24)
\end{aligned}$$

2. if $\nu_2^2 < \nu_1$,

$$\begin{aligned}
S'\left(q_{11}^c\right) &= 2\theta_1, \\
S'\left(q_{21}^c\right) &= \theta_1 + \theta_2 + \frac{\nu_1}{\nu_2}\Delta\theta, \\
S'\left(q_{12}^c\right) &= \theta_1 + \theta_2 + \frac{1}{\nu_2}\Delta\theta, \\
S'\left(q_{22}^c\right) &= 2\theta_2 + \frac{\nu_1}{\nu_2}\Delta\theta. \qquad (25)
\end{aligned}$$

The contract is asymmetric in transfers and quantities ($q_{21}^c > q_{12}^c$) *and there is both underproduction and overproduction with respect to the second-best contract:*

$$\begin{aligned}
q_{11}^c &= q_{11}^{sb}, \\
q_{21}^c &= q_{21}^{sb}, \\
q_{12}^c &< q_{12}^{sb}, \\
q_{22}^c &> q_{22}^{sb}.
\end{aligned}$$

Proof. Any collusion-proof contract satisfies conditions (7) and (9). So, the optimal collusion-proof mechanism solves the following problem

$$\max_{\left(t_{ij}^1, t_{ij}^2, q_{ij}\right)} \sum_i \sum_j \nu_i \nu_j \left[S\left(q_{ij}\right) - t_{ij}^1 - t_{ij}^2\right]$$
subject to
$(7), (8), (9)$.

We do not need to include agent 1's incentive compatibility constraints, because they are implied by (7) and (9). The monotonicity conditions $q_{11} \geq$

$q_{21} \geq q_{12} \geq q_{22}$ have to be satisfied and the binding constraints are

$$\begin{aligned}
t_{11}^2 - \theta_1 q_{11} &\geq t_{12}^2 - \theta_1 q_{12}, \\
t_{21}^2 - \theta_1 q_{21} &\geq t_{22}^2 - \theta_1 q_{22}, \\
t_{12}^2 - \theta_2 q_{12} &\geq 0, \\
t_{22}^2 - \theta_2 q_{22} &\geq 0, \\
t_{22}^1 - \theta_2 q_{22} &\geq 0, \\
t_{11}^1 + t_{11}^2 - 2\theta_1 q_{11} &\geq t_{21}^1 + t_{21}^2 - 2\theta_1 q_{21}, \\
t_{21}^1 + t_{21}^2 - (\theta_1 + \theta_2) q_{21} &\geq t_{12}^1 + t_{12}^2 - (\theta_1 + \theta_2) q_{12}, \\
t_{12}^1 + t_{12}^2 - (\theta_1 + \theta_2) q_{12} - \frac{\nu_1}{\nu_2} \Delta\theta q_{12} &\geq t_{22}^1 + t_{22}^2 (\theta_1 + \theta_2) q_{22} - \frac{\nu_1}{\nu_2} \Delta\theta q_{22}.
\end{aligned}$$

Indeed, if the monotonicity condition is satisfied, all the other constraints are satisfied at the optimal contract.

Solving the system gives the corresponding transfers. Replacing in the principal's objective function we have that the problem writes

$$\max_{(q_{ij})} \left\{ \begin{array}{l}
\nu_1^2 \left[S(q_{11}) - 2\theta_1 q_{11} - \Delta\theta (q_{21} + q_{22}) - \Delta\theta \frac{\nu_1}{\nu_2} (q_{12} - q_{22}) \right] \\
+\nu_1 \nu_2 \left[S(q_{12}) - (\theta_1 + \theta_2) q_{12} - \Delta\theta q_{22} - \Delta\theta \frac{\nu_1}{\nu_2} (q_{12} - q_{22}) \right] \\
+\nu_1 \nu_2 \left[S(q_{21}) - (\theta_1 + \theta_2) q_{21} - \Delta\theta q_{22} - \Delta\theta \frac{\nu_1}{\nu_2} (q_{12} - q_{22}) \right] \\
+\nu_2^2 \left[S(q_{22}) - 2\theta_2 q_{22} \right]
\end{array} \right\}, \tag{26}$$

and maximizing with respect to q_{ij} gives conditions (24). This is indeed a solution if the monotonicity condition is satisfied. It is easy to see that $q_{11}^c > q_{21}^c > q_{12}^c$. However, $q_{12}^c \geq q_{22}^c$ if and only if $\nu_2^2 \geq \nu_1$:

$$S'(q_{22}^c) - S'(q_{12}^c) = \frac{\nu_2^2 - \nu_1}{\nu_2^3} \Delta\theta.$$

So, if $\nu_2^2 < \nu_1$, the monotonicity constraint is binding: $q_{12}^c = q_{22}^c$ and solving problem (26) with the constraint $q_{12} = q_{22}$ gives (25).

$$S'(q_{12}^c) - S'(q_{21}^c) = \left\{ \begin{array}{ll} \frac{\nu_1}{\nu_2^2} \Delta\theta & \text{if } \nu_2^2 \geq \nu_1 \\ \Delta\theta & \text{if } \nu_2^2 < \nu_1 \end{array} \right. > 0,$$

which implies that the contract is asymmetric in quantities. Finally,

$$\begin{aligned}
S'(q_{22}^c) - S'\left(q_{22}^{sb}\right) &= \left\{ \begin{array}{ll} -\frac{\nu_1^2}{\nu_2^3} \Delta\theta & \text{if } \nu_2^2 \geq \nu_1 \\ -\frac{\nu_1}{\nu_2} \Delta\theta & \text{if } \nu_2^2 < \nu_1 \end{array} \right. < 0, \\
S'(q_{12}^c) - S'\left(q_{12}^{sb}\right) &= S'(q_{12}^c) - S'(q_{21}^c) > 0,
\end{aligned}$$

so, $q_{12}^c < q_{12}^{sb}$ and $q_{22}^c > q_{22}^{sb}$. ■

The optimal collusion-proof contract entails no rent for a high-cost agent 2 and the minimum informational rent for a low-cost agent 2. Therefore, nothing is changed for agent 2 with respect to the second-best contract in terms of transfer functions (of course, the effective transfers are different because the quantities are distorted). So, the whole loss coming from the possibility of collusion is transferred to agent 1, in order to prevent him from offering a collusion contract that could undo the principal's offer. Indeed, agent 1 receives a rent *even when he is inefficient*. The rent is equal to 0 when both agents are inefficient, but is strictly positive whenever agent 1 is inefficient and agent 2 is efficient if $q_{12}^c > q_{22}^c$. The reason is that the principal has to guarantee a non-negative rent in all states of nature and, at the same time, avoid collective misreport. The principal wants to discourage a coalition of (θ_1, θ_2) from pretending to be (θ_2, θ_2) (that was the problem with the second-best contract) by increasing agent 1's rent in state (θ_1, θ_2). This implies, then, that a rent has to be given to agent 1 to prevent a coalition of (θ_2, θ_1) from pretending to be (θ_1, θ_2), which is now more attractive.

This additional rent is proportional to $(q_{12} - q_{22})$, so the principal has incentives to decrease production in state (θ_1, θ_2) and increase it in state (θ_2, θ_2), compared to the second-best quantities. In any case, however, production is downward distorted compared to the first-best levels. The distortions with respect to the second-best increase in ν_1. For ν_1 large enough the monotonicity condition $q_{12} \geq q_{22}$ becomes binding and the optimal collusion-proof mechanism entails bunching. When ν_1 is large, it becomes too costly for the principal to deter collusion with a separating contract. The principal would like to offer a contract with $q_{12} < q_{22}$, but this goes against collusion-proofness. Therefore, she destroys the stake of collusion by choosing $q_{12} = q_{22}$.

One interesting result of Proposition 10 is that the principal offers an asymmetric contract, both in transfers and in quantities. The asymmetry in transfers comes from the fact that agents actually are asymmetric with respect to the bargaining power at the collusion stage, so the principal has to give special incentives to agent 1 in order to prevent collusion. The fact that collusion happens before the agents decide whether to accept or reject the grand contract introduces one additional asymmetry, now in quantities, aiming at reducing informational rents in an efficient way. Distorting q_{21} is useful to reduce the rent in state (θ_1, θ_1), while distorting q_{12} helps reducing rents in all states of nature. So the principal is willing to reduce more production in state (θ_1, θ_2) than in state (θ_2, θ_1), even though the total marginal cost is the same in both cases. Indeed, the virtual marginal cost in state (θ_1, θ_2) is $\theta_1 + \theta_2 + \frac{\nu_1}{\nu_2}\Delta\theta$, larger than in state (θ_2, θ_1). The virtual marginal cost includes the cost of the informational rent that has to be given by agent 1 to agent 2 in order to obtain truthful revelation at the collusion stage.

5 Discussion

We considered a very simple model of collusion under asymmetric information. However, many results may be extended to more general cases. According to the informed principal literature, the results concerning the collusion game are also valid when agents have n types. The methodology of analysis can, therefore, be used to look at cases with more than 2 types.

All along this paper, we have looked at a very particular production function that implies strong complementarities between the two agents. In particular, the optimal punishment strategy is highly associated to this property. Indeed, the harshest punishment we can think of when goods are perfect complements is the rejection of the grand contract, because the production of one agent is completely useless without the production of the other. Of course, this is not the case anymore when goods are substitutes. Take, for instance the other extreme of perfect substitutes. Then, the rejection of the grand contract more than a punishment becomes a reward, because in this case the agent is sure that he will be chosen to produce. On the contrary, the harshest punishment is to announce a low cost in order to exclude the other agent from the market. The same is true in an auction, in which the agents compete for the good that is being offered. Nevertheless, the methodology developed here can also be used to analyze these cases, with the corresponding adjustment in the optimal punishment strategy.

One point that deserves some words concerns the enforcement of collusion. In order to be able to apply the tools available in contract theory, we have to assume that there is a third party able to enforce collusion. However, collusion is meant to be a secret (and often illegal) agreement between the parties, so it is difficult to think that a court of justice will play this role. A more complete analysis would incorporate collusion as part of a repeated game in which enforcement comes either from reputation effects or from the threat of punishment in posterior stages of the game. In the same lines of discussion, we have assumed that there is an uninformed third party (the witness) who is used as a commitment device to enforce an ex post suboptimal punishment strategy. Although this third party is quite innocuous, in the sense that is called by the colluding parties, and is uninformed about all relevant variables (it could be randomly chosen from the telephone directory), the participation of a third party goes against the secrecy that involves a collusion agreement. We can imagine colleagues playing the role of third parties as a signal of their willingness to participate of such collusive contracts in the future.

The last important issue is the allocation of the bargaining power at the collusion stage. We considered here the simplest possible case in which one party has all the bargaining power at the collusion stage and the identity of this party is common knowledge. Thus, the principal offers a collusion-proof contract, knowing that it is only agent 1 who can offer a take-it-or-leave-it side contract. Another possibility, certainly more realistic, is that the allocation of the bargaining power is private information of the colluding parties. This case, of course, is much more difficult to deal with because, on top of collusion, the principal faces a problem of multidimensional screening.

6 Conclusions

In this paper we analyzed the problem of collusion under asymmetric information in mechanisms with multi-agents. The main contribution with respect to the existing literature is that we explicitly model collusion as an offer from one informed party to the other one. This introduces more realism in the analysis of coalition formation, at the cost of adding technical complexities in solving the collusion problem. However, we believe that these technical difficulties are worth to be looked at, in order to understand the real constraints that the principal has to consider when collusion is an issue.

The results in this kind of models are usually very sensitive to the timing of the game. For this reason, we look here at two alternative timings. First, we assume that agents decide whether to accept or reject the principal's offer before collusion occurs. If there is no collusion, the agents play non-cooperatively the grand mechanism. In this context, the relevant framework to analyze the collusion problem is to look at an informed principal with common values, because the status quo utility is determined by the expected utility in the grand contract, which depends, in turn, on the private information of all the agents. The problem with this model is that there is in general a plethora of equilibria, and therefore, questions of equilibrium selection become relevant. We show, however, that the optimal contract without collusion is implementable even when agents can collude. Moreover, even if the agent who offers the side contract could commit to punish the other agent if the latter refuses to collude, the principal is still able to implement the second-best contract. However, the implementation of the second-best contract in this case is essentially asymmetric, because the agents themselves are asymmetric, and therefore, special incentives have to be provided to the agent who has the bargaining power at the collusion stage.

In the second part of the paper, we suppose that agents accept or reject the principal's offer after colluding. Then, the principal loses some degrees of freedom because she has to satisfy the ex post participation constraints of the agents. This, in turn, gives more power to the collusion to undo the principal's offer. Moreover, we assume that agent 1 commits to punish agent 2 in the grand mechanism if agent 2 rejects the side contract. Then, if the punishment strategy is independent of agent 1's private information, collusion has to be analyzed using the private value framework, which has, in general, a unique equilibrium and this equilibrium is Pareto optimal. We show that it is ex ante optimal for agent 1 to commit to a type independent punishment strategy. In this context, we show that the second-best contract is no longer implementable when collusion is possible. The principal has too few degrees of freedom to satisfy all the collusion constraints together with the ex post participation constraints. Therefore, collusion becomes costly, and the principal will distort production with respect to the second-best contract in order to trade-off efficiency and rents. She introduces more asymmetries, because the optimal collusion-proof contract is asymmetric both in transfers and in production, and the cost of collusion is transferred to the agent who has the bargaining power at the collusion stage.

In terms of comparisons with the previous literature and, in particular with the third party methodology, our analysis shows that, if agents have differences in their bargaining power, a third party who maximizes joint utility is unable to reproduce the equilibrium of a more realistic collusion game. Indeed, with ex ante collusion and feasible punishment the principal cannot implement the second-best contract when agent 1 has all the bargaining power, while she could have done it if collusion were organized by an uninformed third party maximizing joint utility. Therefore, in order to obtain a better approximation the third pary must be allowed to maximize a weighted sum of the agents' utilities in which the weights will the depend on the differences in bargaining power.

A Appendix

Proof of Proposition 7. Take any offer by the principal: $\left(t_{ij}^1, t_{ij}^2, q_{ij}\right)$. If the punishment strategy is type independent, the relevant framework to analyze collusion is private values. Then, the equilibrium is always Pareto optimal (Maskin and Tirole, 1990 and Quesada, 2002). Suppose that the punishment strategy is to announce θ_1 with probability x and θ_2 with probability $1 - x$. Then, the equilibrium collusion contract solves, for some vector of positive weights (w_1, w_2) problem PV:

$$\max_{(\phi_{ij}, y_{ij})} \sum_i \sum_j w_i \nu_j \left(t^1\left(\phi_{ij}\right) - y_{ij} - \theta_i q\left(\phi_{ij}\right)\right)$$

$$\begin{array}{lr}
\sum_i \nu_i \left(t^2\left(\phi_{ij}\right) + y_{ij} - \theta_j q\left(\phi_{ij}\right)\right) & \\
\geq \sum_i \nu_i \left(t^2\left(\phi_{i\ell}\right) + y_{i\ell} - \theta_j q\left(\phi_{i\ell}\right)\right) \quad \forall j, \forall \ell, & (\lambda_j) \\
\sum_i \nu_i \left(t^2\left(\phi_{ij}\right) + y_{ij} - \theta_j q\left(\phi_{ij}\right)\right) & \\
\geq \overline{u}^2\left(\theta_j\right) \quad \forall j, & (\mu_j) \\
\sum_j \nu_j \left(t^1\left(\phi_{ij}\right) - y_{ij} - \theta_i q\left(\phi_{ij}\right)\right) & \\
\geq \sum_j \nu_j \left(t^1\left(\phi_{kj}\right) - y_{kj} - \theta_i q\left(\phi_{kj}\right)\right) \quad \forall i, \forall k, & (\gamma_i)
\end{array}$$

with

$$\overline{u}^2\left(\theta_j\right) = \max_{\widetilde{\theta}} \left[\begin{array}{c} x\left(t^2\left(\theta_1, \widetilde{\theta}\right) - \theta_j q\left(\theta_1, \widetilde{\theta}\right)\right) \\ +(1-x)\left(t^2\left(\theta_2, \widetilde{\theta}\right) - \theta_j q\left(\theta_2, \widetilde{\theta}\right)\right) \end{array} \right].$$

Moreover, the incentive constraints of agent 1 are not binding at the optimum ($\gamma_1 = \gamma_2 = 0$). Taking derivatives with respect to y_{ij}, we obtain:

$$\begin{aligned}
w_1 &= \lambda_1 - \lambda_2 + \mu_1, \\
\nu_2 w_1 &= -\nu_1 \lambda_1 + \nu_1 \lambda_2 + \nu_1 \mu_2, \\
\nu_1 w_2 &= \nu_2 \lambda_1 - \nu_2 \lambda_2 + \nu_2 \mu_1, \\
w_2 &= -\lambda_1 + \lambda_2 + \mu_2,
\end{aligned}$$

which in turn implies

$$\begin{aligned}
w_1 &= \nu_1\left(\mu_1 + \mu_2\right), \\
w_2 &= \nu_2\left(\mu_1 + \mu_2\right).
\end{aligned}$$

Suppose now that the punishment strategy is to announce θ_1 with probability x_i and θ_2 with probability $1 - x_i$ if agent 1 is of type i. If the punishment strategy is type dependent, the problem has to be analyzed under the common value context. In general, there are many equilibria, but the best possible equilibrium for a type i agent 1 is the allocation that solves

$$\max_{(\phi_{kj}, y_{kj})} \sum_j \nu_j \left(t^1\left(\phi_{ij}\right) - y_{ij} - \theta_i q\left(\phi_{ij}\right)\right)$$

$$\begin{array}{lr}
\sum_k \nu_k \left(t^2\left(\phi_{kj}\right) + y_{kj} - \theta_j q\left(\phi_{kj}\right)\right) & \\
\geq \sum_k \nu_k \left(t^2\left(\phi_{k\ell}\right) + y_{k\ell} - \theta_j q\left(\phi_{k\ell}\right)\right) \quad \forall j, \forall \ell, & (\lambda_j^i) \\
\sum_k \nu_k \left(t^2\left(\phi_{kj}\right) + y_{kj} - \theta_j q\left(\phi_{kj}\right)\right) & \\
\geq \widetilde{u}^2\left(\theta_j\right) \quad \forall j, & (\mu_j^i) \\
\sum_j \nu_j \left(t^1\left(\phi_{kj}\right) - y_{kj} - \theta_k q\left(\phi_{kj}\right)\right) & \\
\geq \sum_j \nu_j \left(t^1\left(\phi_{mj}\right) - y_{mj} - \theta_k q\left(\phi_{mj}\right)\right) \quad \forall m, \forall k, & (\gamma_k^i) \\
\sum_j \nu_j \left(t^1\left(\phi_{kj}\right) - y_{kj} - \theta_k q\left(\phi_{kj}\right)\right) & \\
\geq \widehat{u}^1\left(\theta_k\right) \quad \forall k \neq i, & (w_k^i)
\end{array}$$

with

$$\widetilde{u}^2(\theta_j) = \max_{\widetilde{\theta}} \left[\begin{array}{c} (\nu_1 x_1 + \nu_2 x_2)\left(t^2\left(\theta_1, \widetilde{\theta}\right) - \theta_j q\left(\theta_1, \widetilde{\theta}\right)\right) \\ +(1 - \nu_1 x_1 - \nu_2 x_2)\left(t^2\left(\theta_2, \widetilde{\theta}\right) - \theta_j q\left(\theta_2, \widetilde{\theta}\right)\right) \end{array} \right]$$

and $\widehat{u}^1(\theta_k)$ the RSW payoff of type k agent 1.

Taking derivatives with respect to y_{ij}, we obtain for type 1:

$$\begin{aligned} 1 + \gamma_1^1 - \gamma_2^1 &= \lambda_1^1 - \lambda_2^1 + \mu_1^1, \\ \nu_2\left(1 + \gamma_1^1 - \gamma_2^1\right) &= -\nu_1\lambda_1^1 + \nu_1\lambda_2^1 + \nu_1\mu_2^1, \\ \nu_1 w_2^1 + \nu_1\left(1 - \gamma_1^1 + \gamma_1^2\right) &= \nu_2\lambda_1^1 - \nu_2\lambda_2^1 + \nu_2\mu_1^1, \\ w_2^1 + \left(1 - \gamma_1^1 + \gamma_1^2\right) &= -\lambda_1^1 + \lambda_2^1 + \mu_2^1, \end{aligned} \tag{27}$$

which in turn implies

$$w_2^1 = \mu_1^1 + \mu_2^1 - 2 \geq 0.$$

Similarly for type 2 agent 1, we obtain

$$\begin{aligned} w_1^2 + \left(1 + \gamma_1^2 - \gamma_2^2\right) &= \lambda_1^1 - \lambda_2^1 + \mu_1^1, \\ \nu_2 w_1^2 + \nu_2\left(1 + \gamma_1^1 - \gamma_2^1\right) &= -\nu_1\lambda_1^1 + \nu_1\lambda_2^1 + \nu_1\mu_2^1, \\ \nu_1\left(1 - \gamma_1^1 + \gamma_1^2\right) &= \nu_2\lambda_1^1 - \nu_2\lambda_2^1 + \nu_2\mu_1^1, \\ 1 - \gamma_1^1 + \gamma_1^2 &= -\lambda_1^1 + \lambda_2^1 + \mu_2^1, \end{aligned} \tag{28}$$

and

$$w_1^2 = \mu_1^2 + \mu_2^2 - 2 \geq 0.$$

Therefore, the best possible equilibrium for type i agent 1 can be obtained by solving the following problem CV^i:

$$\begin{array}{ll} \max\limits_{\left(\phi_{ij}, y_{ij}\right)} \sum_i \sum_j w_i \nu_j \left(t^1\left(\phi_{ij}\right) - y_{ij} - \theta_i q\left(\phi_{ij}\right)\right) & \\ \sum_i \nu_i \left(t^2\left(\phi_{ij}\right) + y_{ij} - \theta_j q\left(\phi_{ij}\right)\right) & \\ \geq \sum_i \nu_i \left(t^2\left(\phi_{i\ell}\right) + y_{i\ell} - \theta_j q\left(\phi_{i\ell}\right)\right) \quad \forall j, \forall \ell, & \left(\lambda_j^i\right) \\ \sum_i \nu_i \left(t^2\left(\phi_{ij}\right) + y_{ij} - \theta_j q\left(\phi_{ij}\right)\right) & \\ \geq \overline{u}^2\left(\theta_j\right) \quad \forall j, & \left(\mu_j^i\right) \\ \sum_j \nu_j \left(t^1\left(\phi_{kj}\right) - y_{kj} - \theta_k q\left(\phi_{kj}\right)\right) & \\ \geq \sum_j \nu_j \left(t^1\left(\phi_{mj}\right) - y_{mj} - \theta_k q\left(\phi_{mj}\right)\right) \quad \forall k, \forall m, & \left(\gamma_k^i\right) \end{array}$$

with $(w_1, w_2) = (1, \mu_1^1 + \mu_2^1 - 2)$ if it is type 1 and $(w_1, w_2) = (\mu_1^2 + \mu_2^2 - 2, 1)$ if it is type 2.

Problems PV and CV^i are equivalent if $\widetilde{u}^2(\theta_j) = \overline{u}^2(\theta_j)$, which happens if $x = \nu_1 x_1 + \nu_2 x_2$, and

$$\mu_1^1 + \mu_2^1 = \frac{1+\nu_1}{\nu_1} > 2, \tag{29}$$

$$\mu_1^2 + \mu_2^2 = \frac{1+\nu_2}{\nu_2} > 2. \tag{30}$$

Moreover, (29) and (30) together with (27) and (28) imply that $\gamma_i^i > 0 \; \forall i$, meaning that at least one incentive constraint of agent 1 is binding in problem CV^i. So problem PV is less constrained than problem CV^i. Therefore Agent 1, whatever his type, does strictly better with a type independent punishment strategy, by setting $x = \nu_1 x_1 + \nu_2 x_2$. ■

References

[1] Baliga, S. and T. Sjöström (1998), "Decentralization and Collusion", *Journal of Economic Theory,* 83: 196-232.

[2] Baron, D.P. and D. Besanko (1992), "Information, Control and Organizational Structure", *Journal of Economics & Management Strategy*, 1: 237-275.

[3] Crémer, J. and R. McLean (1985), "Optimal Selling Strategies under Uncertainty for a Discriminant Monopolist when Demands are Interdependent", *Econometrica*, 53: 345-361.

[4] Crémer, J. and R. McLean (1988), "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions", *Econometrica*, 56: 1247-1258.

[5] Demski, J.S. and D.E.M. Sappington (1984), "Optimal Incentive Contracts with Multiple Agents", *Journal of Economic Theory*, 33: 152-171.

[6] Dequiedt, V. (2002), *Optimal Collusion and Optimal Auctions*, Mimeo, University of Toulouse.

[7] Faure-Grimaud, A., J.J. Laffont and D. Martimort (2002), *Collusion, Delegation and Supervision with Soft Information,* Mimeo.

[8] Fudenberg, D. and J. Tirole (1991), *Game Theory*, MIT Press, Cambridge.

[9] Kreps, D. and R. Wilson (1982), "Sequential Equilibrium", *Econometrica,* 50: 863-894.

[10] Laffont, J.J. and D. Martimort (2002), *The Theory of Incentives: The Principal-Agent Model*, Princeton University Press, Princeton.

[11] Laffont, J.J. and D. Martimort (2000), "Mechanism Design with Collusion and Correlation", *Econometrica,* 68: 309-342.

[12] Laffont, J.J. and D. Martimort (1998), "Collusion and Delegation", *Rand Journal of Economics,* 29: 280-305.

[13] Laffont, J.J. and D. Martimort (1997), "Collusion under Asymmetric Information", *Econometrica,* 65: 875-911.

[14] Laffont, J.J. and J. Tirole (1993), *A Theory of Incentives in Procurement and Regulation*, MIT Press, Cambridge.

[15] Maskin, E. and J. Tirole (1990), "The Principal-Agent Relationship with an Informed Principal: The Case of Private Values", *Econometrica,* 58: 379-409.

[16] Maskin, E. and J. Tirole (1992), "The Principal-Agent Relationship with an Informed Principal, II: Common Values", *Econometrica,* 60: 1-42.

[17] Milgrom, P. and J. Roberts (1999), "Coalition-Proofness and Correlation with Arbitrary Communication Possibilities", in D'Aspremont, C. (ed.), *Social Organization and Mechanism Design, proceedings of the Second Francqui Colloquium*, 10-11 June 1996, Brussels, De Boeck and Larcier.

[18] Mookherjee, D. and S. Reichelstein (1992), "Dominant Strategy Implementation of Bayesian Incentive Compatible Allocation Rules", *Journal of Economic Theory*, 56: 378-399.

[19] Myerson, R. (1983), "Mechanism Design by an Informed Principal", *Econometrica,* 51: 1767-1798.

[20] Myerson, R. (1984), "Two-Person Bargaining Problems with Incomplete Information", *Econometrica,* 52: 461-487.

[21] Quesada, L. (2002), *Informed Principal in the Private Value Framework: A Simplified Version*, Mimeo, University of Toulouse.

[22] Segal, I. and M. Whinston (2001), *Robust Predictions for Bilateral Contracting with Externalities*, Mimeo http://www.stanford.edu/~isegal/robust_10-29-01.pdf.